

THE ASTRONOMICAL JOURNAL

FOUNDED BY B. A. GOULD

PUBLISHED BY THE AMERICAN ASTRONOMICAL SOCIETY

VOLUME 64

1959 August ~ No. 1271

NUMBER 6

THE DISTRIBUTION OF EXTRAGALACTIC NEBULAE, III*

BY C. D. SHANE, C. A. WIRTANEN AND ULI STEINLIN

Lick Observatory, University of California, Mt. Hamilton, Calif.

Abstract. Counts of extragalactic nebulae are tabulated for four of the nine areas comprising the program. Contour maps showing the distribution are given. The results of the counts for these four areas, plus those for two others previously published, are used to derive a preliminary mean value of the galactic extinction toward the poles. The amount calculated from these new data is 0.46 mag., which is substantially larger than Hubble's generally accepted value of 0.25 mag.

The counts of extragalactic nebulae on plates taken with the Carnegie 20-inch astrograph of the Lick Observatory have been published for seven of the nine areas into which the sky observable from Mount Hamilton has been divided (Shane and Wirtanen 1954, Paper I; Shane 1956, Paper II). The present paper, number III in the series, gives the results for four additional areas. Table I shows the present status of the work.

TABLE I. STATUS OF COUNTS

	Limits of α	Limits of δ	Remarks
Ia	0 ^h , 6 ^h	-20°, +20°	This paper, Table V
	6 ^h , 12 ^h	-20°, +20°	This paper, Table VI
	12 ^h , 18 ^h	-20°, +20°	<i>A. J.</i> 59, 285, 1954; <i>L. O. Bull.</i> , No. 528
	18 ^h , 24 ^h	-20°, +20°	This paper, Table IV
	0 ^h , 6 ^h	+20°, +60°	<i>A. J.</i> 61, 292, 1956; <i>L. O. Bull.</i> , No. 543
	6 ^h , 12 ^h	+20°, +60°	Counted
I	12 ^h , 18 ^h	+20°, +60°	This paper, Table VII
II	18 ^h , 24 ^h	+20°, +60°	Counted
		> +60°	Counted

The uncorrected counts by square degrees for areas I, II, IV and VII are published in this paper as Tables IV to VII inclusive. Plates in each 5° zone of declination are tabulated as separate groups. The first line above each block of the 36 counts on a single plate gives the plate number and the right ascension of the plate center. The next line gives the initial of the observer who counted the plate, S for Shane and W for Wirtanen, followed by a small letter designating

the plate emulsion, and finally the complete plate factor. A detailed description of the method used in making and reducing the counts is given in Paper I.

The reduced counts were obtained by multiplying each of the tabulated values, first by a field correction factor taken from Table II, second by an extinction factor whose logarithm equals 0.18 (sec $z - 1$), and finally by the complete plate factor for the given plate. It is not worthwhile to list here the provisional values of the emulsion and personal factors used in calculating the complete plate factors, since these will be recomputed later on the basis of all nine areas.

The reduced counts were averaged by fours and used to draw the contour maps, Figures 1 to 4, which show the distribution of the extragalactic nebulae.

The discussion of the material in this paper is limited to a preliminary determination of the galactic extinction, and is based on these nebular counts in their present degree of completeness.

In his classic paper on "The Distribution of the Extragalactic Nebulae," Hubble (1934) fitted his counts to the equation

$$\log N = A + B \csc b, \quad (1)$$

where N is the number of nebulae per square degree, b is the galactic latitude taken with a

* *Lick Observatory Bulletin*, No. 562.

TABLE II. FIELD CORRECTIONS

Area I					
1.12	1.04	1.03	1.07	1.16	1.34
1.01	0.96	0.98	1.02	1.06	1.20
0.96	0.95	0.98	1.02	1.06	1.15
0.96	0.95	0.98	1.02	1.06	1.15
1.01	0.96	0.98	1.02	1.06	1.20
1.12	1.04	1.03	1.07	1.16	1.34
Area II					
1.17	1.07	1.05	1.08	1.19	1.38
1.04	0.97	0.98	1.02	1.07	1.23
0.98	0.95	0.98	1.02	1.05	1.16
0.98	0.95	0.98	1.02	1.05	1.16
1.04	0.97	0.98	1.02	1.07	1.23
1.17	1.07	1.05	1.08	1.19	1.38
Area IV					
1.12	1.04	1.02	1.06	1.15	1.33
1.00	0.95	0.98	1.02	1.06	1.19
0.95	0.95	0.98	1.02	1.05	1.13
0.95	0.95	0.98	1.02	1.05	1.13
1.00	0.95	0.98	1.02	1.06	1.19
1.12	1.04	1.02	1.06	1.15	1.33
Area VII					
1.15	1.06	1.04	1.08	1.17	1.37
1.02	0.97	0.98	1.02	1.07	1.21
0.97	0.95	0.98	1.02	1.05	1.15
0.97	0.95	0.98	1.02	1.05	1.15
1.02	0.97	0.98	1.02	1.07	1.21
1.15	1.06	1.04	1.08	1.17	1.37

positive sign, and A and B are constants determined empirically. This equation is based theoretically on an assumed large scale distribution of obscuring matter arranged symmetrically on either side of the galactic plane in uniform layers of infinite extent. Such a distribution of obscuring matter obviously does not closely approximate the real situation. Equation (1) is therefore to be regarded as an interpolation formula whose usefulness depends on the degree to which it fits the observations. When Hubble applied it to his data, the fit was good and yielded a value for B of -0.15 , with no significant difference between separate determinations from the data north and south of the galactic plane. If Hubble's assumptions that for this purpose it is sufficiently accurate to neglect redshift effects, and that the nebulae are uniformly distributed in depth, then his analysis gave an extinction coefficient of $\frac{-B}{0.60}$, or 0.25 mag. for the optical half-thickness

of the absorbing layer. This value has been widely used since the publication of Hubble's paper. The divisor 0.60 comes from the equation

$$\Delta \log N = 0.60 \Delta m, \quad (2)$$

where $\Delta \log N$ is the change in $\log N$ corresponding to a change Δm in limiting magnitude. In a

subsequent paper Hubble (1936) found empirically that the factor should be 0.50 rather than 0.60 . On this basis the extinction coefficient should have been increased to 0.30 mag.

In view of the empirical nature of equation (1) there is no assurance that A and B are independent of galactic longitude. Hubble's counts were not extensive enough to investigate in detail a possible dependence, although he briefly commented on this possibility. The much more complete counts of the current study should permit separate determinations of A and B for different longitude intervals. Values of B for certain 20° longitude intervals were published in Papers I and II of this series. With the exception of a very poorly determined value for $B = +0.05$, they range from -0.13 to -0.37 , with an unweighted mean of -0.37 . This value is more than twice that derived by Hubble, but it is strongly affected by several large values from areas near the longitude of the central galactic bulge. Nevertheless, it seems probable that Hubble's value of B should be revised upward.

To determine B for a given longitude interval with sufficient accuracy, the counts should cover a large range of latitude. This requirement cannot generally be fulfilled with the material available to date, since the full latitude range usually extends into two or more of our nine areas. As shown below, counts of the different areas extend to somewhat different limiting magnitudes that are not well determined. At present, therefore, it is impossible to combine counts in the different areas in a solution for B . In this paper each determination must be limited to data from a particular area, but a more definitive reduction will be made from all areas when the counts have been completed and reduced to a common limiting magnitude.

The data used in the solutions were the mean values of the corrected counts for each plate with its center in the area for which the solution was made. In forming the means, the four corner square degrees were given weight $1/4$, and the sixteen along the edges weight $1/2$, as compared with unity for the sixteen central square degrees. This weighting was used because, to a fair approximation, the corner square degrees are counted on four plates and the other edge square degrees on two. Thus the overweighing of certain square degrees due to the overlaps is largely avoided.

The plate factor, an average field factor, and the zenith extinction corrections were applied

the counts, and the log of the mean N for each plate together with the cosecant of the galactic latitude of the plate center were substituted in equation (1). Least squares solutions for the six areas thus far reduced were made from these observation equations.

Two sets of solutions were carried out. In the first set only plates with $|b| > 39^\circ$ were used.

In the second set all plates with mean $N > 10$ were included. The first set of solutions avoids the strong absorbing clouds in low latitude, but it has small weight for the determination of B because of the restricted range in $\sec b$. With a small range of $\sec b$, the large-scale irregularities in the distribution of the extragalactic nebulae, as well as possible obscuring areas in high latitudes, may seriously affect the derived value of B . On the other hand, $A + B$, which presents the logarithm of the number of nebulae extrapolated to the pole, should be fairly accurate.

In the second set of solutions, conspicuous dark clouds of obscuration in low latitudes should have a disturbing effect on some of the results, but to omit the regions of their occurrence would tend to introduce bias. These second-set solutions seem, on the whole, to be considerably more accurate for determining the galactic extinction, and to be equally reliable with the first set for determining $A + B$.

The limiting photographic magnitude based on Hubble's scale was derived by comparing our counts with Hubble's as explained in Paper I of this series. It is known that this scale has substantial errors, but there is no evidence suggesting that it is not consistent over the sky. On the other hand, our areas were in general counted in sequence, though not in the order of their numerical designations. It is therefore reasonable to expect the counts in them to have different limiting magnitudes due to changing personal equations.

The values of B and $A + B$ for the six areas together with the limiting magnitudes are contained in Table III. The letter "n" or "s" in the first column indicates that the area is predominantly north or south of the galactic equator. The values " $(A + B)$ corr." in the fifth and eighth columns are derived from their immediately preceding columns by reduction to the mean limiting photographic magnitude of 18.19. The other columns are self-explanatory.

The large range in B calculated from latitudes above 39° is evidence of the low weight of these determinations. The average weight of the five values, as computed formally in the course of the least squares solutions, is 2, which may be compared with 31 as the average weight for the B 's in the sixth column. These formal weights do not, however, give a correct impression of the accuracy. There are large scale irregularities in the apparent distribution of the nebulae that can strongly affect B as calculated over a limited portion of the sky. Perhaps the most striking instance in high latitudes is a region of low counts around the north galactic pole and extending toward lower latitudes in the direction of longitude 90° . The Coma cluster of nebulae lies in this relatively sparse area. Other regions of low as well as of abnormally high counts are readily found on the charts.

The values of B in the sixth column, determined for all latitudes higher than the $N = 10$ contour, are more consistent, although the range is still large. An area such as III, which includes the galactic bulge, would be expected to yield a high value.

It is interesting to note that the mean B 's for north and south latitudes are nearly the same. The small excess for north latitudes is easily accounted for by the inclusion of Area III with no compensating area in south latitudes near the galactic bulge. There is thus no evidence from the galactic extinction coefficients that the sun

TABLE III. VALUES B AND $A + B$

Area	Lim. mag.	$ b > 39^\circ$			$ b > 0^\circ$		
		B	$A + B$	$(A + B)$ Corr.	B	$A + B$	$(A + B)$ Corr.
I s	18.24	-0.44	1.82	1.79	-0.25	1.77	1.74
II n	18.02	-0.24	1.81	1.90	-0.21	1.77	1.86
III n	18.30	-0.32	1.88	1.82	-0.42	1.88	1.82
IV s	18.04	-0.04	1.62	1.70	-0.26	1.67	1.75
V s	18.23				-0.14	1.81	1.79
VII n	18.30	-0.05	1.71	1.65	-0.19	1.74	1.68
Mean, N lat.	—	-0.20	1.80	1.79	-0.27	1.80	1.79
Mean, S lat.	—	-0.24	1.72	1.74	-0.22	1.75	1.76
Mean, all lat.	18.19	-0.22	1.77	1.77	-0.24	1.77	1.77

TABLE IV. AREA IV. UNCORRECTED COUNTS OF NEBULAE PER SQUARE DEGREE

[illegible]

TABLE IV. (continued)

$\delta = -10^\circ$	499	18 ^h 00 ^m	931	18 ^h 20 ^m	1265	18 ^h 40 ^m	515	19 ^h 00 ^m	
	Sa	0.957	Wc	0.734	Wf	0.815	Sa	0.957	
	1	1	1	0	0	0	0	0	
	0	1	0	0	0	0	1	0	
	0	0	0	0	0	1	2	0	
	0	0	0	0	0	1	1	0	
	0	1	0	0	0	0	1	0	
	0	0	0	0	0	1	0	0	
864	19 ^h 20 ^m	1288	19 ^h 40 ^m	1286	20 ^h 00 ^m	895	20 ^h 20 ^m	930	20 ^h 40 ^m
Sb	1.000	Sf	0.836	Sf	1.034	Sb	1.367	Wc	0.544
4	6	2	0	5	0	5	3	9	11
4	3	2	0	1	1	11	15	13	17
1	1	0	1	0	0	9	7	31	6
1	0	0	0	1	0	6	7	9	12
2	5	3	2	1	0	10	12	12	3
2	1	1	5	0	1	5	9	6	8
1701	21 ^h 00 ^m	557	21 ^h 20 ^m	1725	21 ^h 40 ^m	1289	22 ^h 00 ^m	1325	22 ^h 20 ^m
Si	0.956	Sa	1.129	Si	1.161	Wf	0.753	Sf	1.043
16	12	29	31	32	9	28	70	52	59
16	18	44	25	27	11	64	69	56	87
23	31	32	35	24	10	69	85	78	135
17	30	31	17	22	8	112	108	121	96
16	25	15	14	18	7	50	54	57	82
19	23	27	20	8	7	48	76	62	53
1285	22 ^h 40 ^m	1747*	23 ^h 00 ^m	1001	23 ^h 20 ^m	662	23 ^h 40 ^m	1086	0 ^h 00 ^m
Wf	0.993	Si	0.763	Wd	1.065	Wb	0.724	We	0.648
17	25	31	28	42	12	37	49	64	44
30	36	47	40	26	39	37	29	53	44
41	33	36	27	33	45	29	30	34	26
55	18	29	23	29	25	81	32	30	40
42	41	28	28	17	33	59	35	29	52
19	19	13	28	36	26	36	45	27	41
$\delta = -5^\circ$	509	18 ^h 00 ^m	918	18 ^h 20 ^m	1656	18 ^h 40 ^m	558	19 ^h 00 ^m	
	Sa	0.914	Wb	0.767	Wi	0.780	Wa	0.734	
	0	0	0	0	0	0	0	2	
	0	0	0	0	0	0	1	0	
	1	0	0	0	0	0	0	0	
	1	0	0	1	0	1	0	0	
	0	1	0	0	0	1	0	0	
	0	0	0	0	0	0	1	2	
1315	19 ^h 20 ^m	1678	19 ^h 40 ^m	1290	20 ^h 00 ^m	1269	20 ^h 20 ^m	932	20 ^h 40 ^m
Sf	1.062	Si	1.016	Sf	1.062	Sf	0.985	Sc	1.519
0	1	1	0	0	0	13	8	13	5
1	2	0	0	0	0	7	15	6	7
0	0	0	1	0	2	10	9	7	2
2	1	0	0	0	0	11	6	11	6
2	2	1	0	0	2	4	11	8	9
2	0	1	0	1	1	6	3	9	9
1693	21 ^h 00 ^m	567	21 ^h 20 ^m	1347	21 ^h 40 ^m	1328	22 ^h 00 ^m	1291	22 ^h 20 ^m
Wi	0.602	Sa	1.296	Sg	1.072	Sf	1.227	Wf	0.678
32	33	19	28	21	23	29	32	21	24
32	33	21	30	30	37	18	9	17	15
15	24	30	45	42	23	30	25	24	35
25	29	26	41	33	15	17	32	57	41
21	27	33	34	24	8	24	29	52	30
31	30	31	43	39	19	20	66	41	52
1739	22 ^h 40 ^m	1430	23 ^h 00 ^m	1714	23 ^h 20 ^m	568	23 ^h 40 ^m	1320	0 ^h 00 ^m
Si	1.277	Wg	0.896	Si	0.849	Wa	0.846	Wf	0.903
17	15	25	15	18	15	39	73	47	33
20	50	33	23	33	13	55	38	35	45
41	51	30	20	18	9	43	41	72	43
37	46	73	12	27	22	35	32	50	33
24	24	30	31	15	26	30	36	49	42
20	18	23	23	28	10	31	38	58	32

TABLE IV. (continued)

$\delta = 0^\circ$		514 S a	18 ^h 00 ^m 0.957	924 W b	18 ^h 20 ^m 0.767	1696 W i	18 ^h 40 ^m 0.780	520 S a	19 ^h 00 ^m 0.957
		0 0 3	1 3 0	1 0 0	2 0 1	0 1 0	0 0 1	0 1 1	0 0 0
		1 1 1	1 2 1	1 0 2	0 0 1	0 2 0	0 0 1	0 0 0	1 0 0
		0 0 0	0 2 2	0 0 0	0 0 0	2 2 0	0 0 0	0 0 0	0 0 0
		1 0 1	2 2 1	1 0 0	0 1 2	0 0 0	1 0 0	0 0 0	0 0 0
		0 0 1	0 1 1	1 0 0	0 2 1	1 1 0	0 0 0	1 0 0	0 0 0
		0 0 0	0 1 0	0 0 0	1 0 0	0 0 0	0 0 0	2 0 1	0 2 0
510 W a	19 ^h 20 ^m 0.734	1276 S f	19 ^h 40 ^m 1.062	1324 S f	20 ^h 00 ^m 1.062	1281 S f	20 ^h 20 ^m 1.006	1357 S g	20 ^h 40 ^m 0.958
2 0 0	2 1 0	2 2 2	1 4 0	9 10 1	3 5 1	14 11 18	7 7 6	7 5 7	9 7 7
4 0 0	2 2 3	3 1 0	3 0 1	10 4 14	11 6 1	8 12 13	6 4 6	7 4 9	14 19 16
1 1 0	0 1 6	1 2 3	0 1 2	4 12 5	4 0 1	4 11 5	12 2 6	13 21 11	12 3 6
0 2 1	0 0 2	1 0 2	1 1 0	7 7 2	4 5 1	3 13 8	20 13 5	27 13 8	10 6 5
1 2 1	0 0 2	2 0 2	1 1 1	3 12 5	0 4 1	11 24 4	16 6 2	23 29 24	13 7 13
1 1 1	0 3 5	1 1 0	0 0 1	3 4 4	3 5 3	14 10 8	4 2 2	24 10 26	14 17 17
2014 S k	21 ^h 00 ^m 1.276	570 S a	21 ^h 20 ^m 0.892	1004 S d	21 ^h 40 ^m 1.110	1277** S f	22 ^h 00 ^m 2.255	1331 S f	22 ^h 20 ^m 1.205
43 7	6 13 11 7	18 32	29 29 31 56	38 14	16 16 16 23	10 6	22 12 6 18	25 20 35	30 35 17
17 13	7 14 6 9	29 45	48 39 19 20	10 12	21 13 30 14	15 21	27 14 15 4	28 29 23	24 19 22
12 23	20 16 11 12	38 42	41 26 28 13	20 33	19 32 47 19	16 16	21 12 10 6	46 38 25	19 18 23
36 28	24 30 25 20	48 52	27 19 39 34	19 18	15 31 28 22	17 12	17 15 8 6	23 48 26	11 28 19
26 24	19 8 12 8	28 45	8 34 18 27	20 23	30 24 21 24	9 10	8 10 10 9	28 35 47	27 20 14
24 32	15 24 7 10	30 20	10 19 19 20	23 31	28 29 17	6 11	3 16 4 4	20 24 65	27 30 20
1428 W g	22 ^h 40 ^m 0.577	1717 S i	23 ^h 00 ^m 1.057	1775 W i	23 ^h 20 ^m 0.959	654 W b	23 ^h 40 ^m 0.700	981** W d	0 ^h 00 ^m 1.483
41 44	39 42 49 51	19 21	26 30 17 11	27 18	30 32 31 26	54 36	38 33 34 37	24 25 16	23 22 22
40 60	73 40 122 47	14 17	37 29 31 18	27 40	33 49 31 18	52 60	35 24 59 38	28 21 32	20 30 22
79 46	105 48 54 87	22 24	33 31 39 51	29 19	28 31 39 18	38 32	40 29 50 40	40 30 54	38 36 22
38 40	49 50 34 44	37 44	20 27 38 28	42 20	25 23 27 30	42 20	48 64 63 59	42 42 23	22 23 13
42 27	59 89 55 44	25 34	15 20 31 23	44 34	28 37 34 22	48 33	32 37 59 80	31 42 20	63 14
42 39	62 34 44 35	32 22	20 18 37 18	55 72	43 50 80 42	33 75	40 20 23 41	18 34 38	30 33 26
$\delta = +5^\circ$		913 S b	18 ^h 00 ^m 1.000	1261 W f	18 ^h 20 ^m 0.813	1700 W i	18 ^h 40 ^m 0.780	566 W a	19 ^h 00 ^m 0.734
		4 2 3	2 2 1	1 1 0	1 3 8	1 0 0	1 1 0	0 0 0	0 0 0
		2 6 1	3 1 6	1 4 1	6 2 5	0 1 0	2 1 0	0 0 0	0 0 0
		2 2 2	2 2 3	5 1 0	3 2 2	0 0 0	0 0 0	0 0 0	0 0 1
		3 4 4	1 0 2	2 0 0	0 2 3	0 0 0	0 0 0	0 0 0	0 0 0
		2 0 1	3 4 0	5 1 1	1 2 1	1 0 1	1 2 0	0 0 0	0 0 0
		5 1 4	4 3 3	2 3 0	0 0 0	0 1 0	0 1 1	0 0 0	0 0 0
1318 S f	19 ^h 20 ^m 1.062	1321 W f	19 ^h 40 ^m 0.813	1344 S g	20 ^h 00 ^m 0.824	1284 W f	20 ^h 20 ^m 1.140	1969 S k	20 ^h 40 ^m 0.911
0 0 0	2 0 0	0 0 1	0 0 0	5 6 7	0 2 0	6 3 6	11 4 5	16 22 11	12 6 7
0 0 0	0 0 0	0 1 0	1 1 2	5 4 10	3 1 0	6 1 9	8 5 2	19 9 16	12 6 4
1 1 0	0 0 0	5 3 1	0 0 1	5 8 1	3 9 2	6 4 5	5 4 7	8 15 28	6 5 5
0 0 0	0 2 0	11 4	2 0 3	7 9 1	1 3 4	10 4	5 4 5 4	7 8 13	4 8 10
0 0 1	0 1 1	2 7 1	2 0 1	13 13	3 1 0	5 7 8	5 3 6	13 17 6	11 10 4
2 0 1	1 0 1	0 4 3	4 7 3	8 6 1	1 2 2 1	14 11	15 2 6 9	9 9 16	12 7 14
1690 S i	21 ^h 00 ^m 1.013	649 S b	21 ^h 20 ^m 0.883	546 S a	21 ^h 40 ^m 0.886	1027 W d	22 ^h 00 ^m 1.208	1731 S i	22 ^h 20 ^m 0.953
14 25	17 28 7 14	23 33	23 18 25 21	40 25	22 19 27 23	20 14	15 15 17 23	11 18 20	18 21 27
20 19	22 14 12 26	21 43	27 37 30 22	24 26	35 39 33 22	26 15	12 22 33 10	21 25 20	23 11 22
17 19	27 14 9 7	33 41	34 30 29 19	22 18	27 32 34 39	18 4	19 13 14 11	30 22 23	25 19 13
23 21	12 12 13 13	47 50	56 20 23 25	41 41	29 39 20 34	25 20	25 13 20 24	19 26 31	19 18 24
14 16	13 13 12 7	85 68	50 36 18 23	59 23	10 16 23 39	13 20	14 14 21 41	25 26 38	15 25 15
68 21	12 18 12 12	28 46	40 41 31 45	53 22	27 19 19 29	15 14	27 16 14 37	30 21 51	27 26 12
1770 S i	22 ^h 40 ^m 1.304	1720 W i	23 ^h 00 ^m 0.816	1778 S i	23 ^h 20 ^m 0.954	1427 W g	23 ^h 40 ^m 1.091	1726 W i	0 ^h 00 ^m 0.704
22 24	20 29 9 6	64 87	44 51 29 25	22 24	24 47 42 33	26 30	14 21 30 29	21 31 46	68 33 45
24 20	25 25 13 9	53 42	32 89 34 34	21 33	47 39 50 49	67 37	8 19 24 20	42 62	57 55 38 71
44 36	19 32 28 19	47 58	30 41 26 53	26 21	26 26 35 39	48 39	31 25 21 20	54 58	74 79 74 74
27 44	25 21 23 9	23 51	24 30 39 35	31 25	38 33 30 17	16 17	22 21 26 23	56 77	92 49 41 34
51 33	40 20 19 27	49 37	40 42 51 47	35 44	27 34 29 35	35 29	24 24 23 34	58 79	72 73 71 59
20 30	24 27 32 22	31 25	28 31 16 20	23 17	38 38 34 17	45 25	36 15 29 21	49 56	48 42 42 51

TABLE IV (continued)

$\delta = +10^\circ$																																							
915 18 ^h 00 ^m					1654 18 ^h 20 ^m					1692 18 ^h 40 ^m					877 19 ^h 00 ^m																								
S b 1.155					W i 0.779					W i 0.779					S b 1.000																								
7 6 6 5 10 5					3 4 6 0 10 6					0 0 1 0 0 1					0 0 3 2 3 2																								
6 6 13 3 6 2					2 6 3 2 9 7					0 0 1 0 1 1					0 0 0 2 0 1																								
5 7 11 15 2 3					1 4 2 1 2 3					1 1 0 0 3 0					0 1 0 2 0 0																								
2 12 5 2 2 1					3 0 1 2 0 3					0 0 0 0 0 1					0 1 0 0 2 2																								
1 5 3 1 5 1					2 1 0 1 1 1					1 1 0 0 0 0					0 0 1 0 0 0																								
2 0 2 2 5 0					5 1 5 3 6 2					1 1 0 0 0 0					1 0 0 2 0 0																								
882 19 ^h 20 ^m					1345 20 ^h 00 ^m					2011 20 ^h 20 ^m					1365 20 ^h 40 ^m																								
W b 0.767					W g 0.832					S k 0.851					W g 1.144																								
1 1 0 2 0 0					3 2 1 0 2 0					7 3 7 1 2 3					2 6 15 10 6 5																								
0 0 0 0 0 0					2 0 1 0 0 0					6 6 5 1 2 2					11 14 10 6 4 9																								
0 1 0 0 0 0					4 3 1 1 0 1					9 14 10 2 2 4					11 10 9 24 19 5																								
0 0 0 0 0 0					4 1 1 0 0 0					16 4 9 4 2 4					11 12 14 12 9 17																								
0 0 0 0 0 0					2 2 0 1 1 4					9 9 8 4 5 2					6 12 7 16 9 8																								
0 0 0 1 0 0					7 2 6 0 2 1					9 6 6 11 6 6					8 17 18 16 4 10																								
959 21 ^h 00 ^m					993 21 ^h 40 ^m					1024 22 ^h 00 ^m					1346 22 ^h 20 ^m																								
S d 0.877					S d 1.174					S d 0.855					W g 0.826																								
4 26 34 19 17 9					26 19 14 21 21 23					26 23 26 14 22 23					34 19 34 41 45 32																								
0 25 39 21 21 13					13 14 17 17 33 40					34 21 13 21 13 8					55 27 50 28 32 36																								
7 15 20 31 18 9					14 23 22 57 39 41					18 26 21 18 11 12					21 32 43 29 34 27																								
2 17 18 20 20 11					31 27 44 31 35 48					31 46 35 25 28 25					31 36 29 31 42 35																								
0 27 30 26 17 8					51 31 19 27 26 28					40 44 50 44 52 50					20 30 33 40 37 25																								
7 28 33 29 12 18					32 30 23 18 31 11					26 22 35 40 30 26					17 21 29 27 34 33																								
676 22 ^h 40 ^m					1760 23 ^h 20 ^m					691 23 ^h 40 ^m					1780 0 ^h 00 ^m																								
W b 0.630					W i 0.807					W b 0.592					W i 0.889																								
2 44 58 43 48 30					37 34 62 45 33 31					44 31 45 29 28 39					28 45 45 60 49 25																								
5 93 48 59 68 52					32 37 44 50 33 34					88 32 56 30 51 43					29 40 30 27 61 50																								
2 74 67 50 57 23					21 38 30 52 59 44					105 43 39 47 39 47					49 38 48 27 29 59																								
7 36 46 41 32 23					56 72 67 62 119 61					62 75 61 38 66 74					34 29 27 13 31 45																								
1 48 71 49 23 21					43 65 55 73 43 46					72 84 137 74 74 42					30 31 51 51 42 37																								
6 59 52 55 28 27					43 39 41 72 70 55					55 49 28 46 46 48					35 30 45 56 32 38																								
$\delta = +15^\circ$																																							
1280 18 ^h 00 ^m					1686 18 ^h 20 ^m					1689 18 ^h 40 ^m					879 19 ^h 00 ^m																								
S f 0.852					W i 1.037					W i 0.779					W b 0.767																								
2 9 6 9 6 15					1 2 2 5 6 3					3 0 3 1 8 2					0 0 0 0 1 0																								
6 18 5 4 9 25					2 5 2 5 1 4					0 3 0 2 3 6					0 0 0 0 0 0																								
8 7 11 9 11 17					1 1 6 5 12 5					2 2 0 0 1 1					0 0 0 0 0 0																								
13 7 8 14 13 27					0 2 2 10 9 10					1 0 0 1 1 1					0 0 0 0 0 0																								
15 4 10 13 17 10					1 2 1 5 8 11					0 0 1 1 0 2					1 0 0 1 0 0																								
6 9 8 9 14 3					0 2 3 1 3 2					3 1 2 0 0 1					0 0 0 0 0 0																								
857 19 ^h 20 ^m					1665 20 ^h 00 ^m					1349 20 ^h 20 ^m					1687 20 ^h 40 ^m																								
S b 1.000					S i 1.016					W g 0.839					S i 1.082																								
1 1 0 0 0 0					1 2 0 0 0 1					4 1 5 1 3 1					7 10 1 1 6 2																								
0 0 1 0 0 0					1 0 1 0 0 0					2 2 4 5 4 3					13 19 5 3 6 3																								
1 0 1 0 0 0					1 2 0 0 1 0					5 4 8 5 0 2					7 5 4 3 1 5																								
0 0 1 0 0 0					1 2 0 1 0 0					7 7 5 4 2 2					4 5 2 4 7 8																								
0 0 0 0 0 0					3 1 2 3 0 0					10 10 4 8 3 2					8 8 10 5 10 6																								
0 1 0 0 0 1					2 1 2 0 1 1					10 7 9 2 0 1					4 8 15 14 10 7																								
956 21 ^h 00 ^m					943 21 ^h 40 ^m					963 22 ^h 00 ^m					1032 22 ^h 20 ^m																								
S d 0.783					S c 0.825					W d 1.222					S d 0.960																								
2 35 18 26 18 11					23 22 16 26 19 11					24 23 14 18 12 9					34 33 37 27 24 29																								
8 15 14 19 25 21					26 20 16 21 32 11					57 34 49 14 19 17					35 48 38 25 43 49																								
2 40 36 35 20 14					47 43 21 24 25 25					30 31 24 19 22 38					33 29 34 24 40 23																								
2 39 37 21 18 14					36 27 35 41 29 20					46 34 67 31 26 25					40 41 38 47 54 32																								
3 37 33 37 32 15					33 38 44 38 36 23					60 31 20 23 10 26					33 34 26 31 75 48																								
8 28 56 33 25 13					32 36 39 41 38 16					25 19 23 13 7 19					22 21 24 36 32 27																								
686 22 ^h 40 ^m					693* 23 ^h 20 ^m					942 23 ^h 40 ^m					944 0 ^h 00 ^m																								
W b 0.763					W b 0.609					W c 0.995					W c 0.866																								
8 59 56 33 56 30					55 57 114 42 31 47					52 54 31 27 31 25					43 38 45 43 34 62																								
8 51 75 38 26 30					53 72 132 61 66 49					70 51 32 42 32 28					32 37 48 40 47 72																								
7 59 48 40 41 38					31 54 93 76 60 70					44 33 32 35 53 20					55 49 73 60 62 63																								
4 55 42 44 49 48					88 79 179 46 62 76					40 45 42 61 36 36					50 58 36 63 57 40																								
9 42 33 53 52 42					50 74 90 50 79 59					56 35 46 37 24 22					39 49 60 57 68 59																								
4 49 63 52 69 37					53 64 71 48 38 52					36 26 30 20 26 29					28 46 38 53 52 49																								

TABLE IV. (continued)

<

is unsymmetrically placed with reference to the distribution of obscuring matter north and south of the galactic plane.

The quantity $A + B$ represents the log number of extragalactic nebulae per square degree reduced to the pole, with the corrected values in the last column of Table III in reasonable agreement for the two poles. The observed excess for the north as compared with the south pole is too small to be very significant. It is well known that for the brighter extragalactic nebulae the north polar cap is more densely populated than the south polar cap. According to Shapley (1957), however, the surface densities of nebulae between 19th and 20th magnitude at the two poles are about equal. Hubble's counts, which were extrapolated by him to magnitude 20, and which actually extended on the average only to approximate magnitude 18.9, indicate an excess of galaxies in the south polar cap of about 7 per cent. The evidence at hand gives no significant indication of any difference between the densities of the fainter nebulae in the two hemispheres.

One further point should be mentioned. In calculating the effect of atmospheric extinction on $\log N$, we assumed that a difference of 1.0 in

limiting magnitude corresponds to 0.60 in $\log N$. Hubble likewise made this assumption, which neglects the effect of redshift. Hubble's empirical determination of the relation between limiting magnitude and $\log N$ yielded a coefficient of 0.50 instead of 0.60. With this smaller value we calculated corrections to B and $A + B$ in Table III. In no instance did the change in B or $A + B$ exceed 0.01. The effect of using 0.60 instead of 0.50, or some more probable intermediate value, may therefore be neglected in correcting for the effect of atmospheric extinction.

The mean galactic extinction at the two poles is found by dividing B by a factor that depends on the allowance for redshift as based on limiting magnitude. In the present instance the observed limiting magnitude of the plates for nebulae averaged 18.2, but reduction to the zenith decreased the average by about 0.1 mag., and reduction to the galactic poles by another 0.2 mag. In the average we should thus use a redshift corresponding to 17.9 mag., or a value of $d\lambda/\lambda = 0.077$. The corresponding quantity by which we divide B is 0.52. The limiting magnitude 17.9 is affected by redshift. If there were no redshift, a nebula of this apparent brightness would have

TABLE V. AREA I, UNCORRECTED COUNTS OF NEBULAE PER SQUARE DEGREE

$\delta = -20^{\circ}$	977 $0^h 00^m$		1025 $0^h 20^m$		1326 $0^h 40^m$		659 $1^h 00^m$		
	Wd 0.927		Sd 1.418		Wf 0.998		Sb 1.052		
	13 26 27 33 26 23		15 18 18 29 13 14		31 47 41 31 30 11		55 44 33 50 30 25		
	26 24 40 23 42 40		32 16 28 16 16 10		39 36 83 44 25 20		38 40 37 44 30 23		
	40 20 26 17 43 23		27 18 32 35 37 50		68 85 55 47 44 35		37 33 48 49 60 50		
	27 37 21 36 42 33		24 22 46 19 23 31		44 58 48 39 39 25		21 30 48 36 34 30		
	18 30 28 40 32 25		25 27 46 29 12 9		50 74 47 58 48 35		23 44 44 49 45 40		
	26 22 26 25 29 25		21 25 27 36 10 14		42 44 79 85 37 32		18 25 64 68 50 39		
	975 $1^h 20^m$		663 $2^h 00^m$		1795 $2^h 20^m$		1082 $2^h 40^m$		
	Wd 1.273		Sb 0.910		Sj 1.041		Se 0.663		
28 27 33 58 36 39	1759 $1^h 40^m$		27 29 40 29 34 30		43 54 60 32 23 27		36 77100 66 35 72		
17 28 43 34 30 25	Si 1.026		53 46 29 26 25 41		25 19 31 37 29 38		38 43 41 50 48 27		
24 44 28 29 11 31	47 21 24 13 14 28		30 36 26 20 20 34		23 55 32 17 24 17		25 54 57 78 91 35		
13 30 32 32 19 19	29 11 17 28 19 19		24 22 23 40 34 48		57 37 30 20 20 21		53 45 38 69 43 76		
28 33 21 38 17 23	43 35 25 17 15 18		17 27 14 42 34 37		29 39 31 43 37 15		46 46 38 36 26 31		
15 30 23 33 21 20	45 35 32 37 31 56		28 44 30 25 27 29		14 49 14 15 10 25		17 51 32 31 21 41		
716 $3^h 00^m$		1040 $3^h 40^m$		1800 $4^h 00^m$		664 $4^h 20^m$			
Sb 0.956		Sd 1.321		Wj 0.599		Sb 1.182			
56 43 48 34 33 48	660 $3^h 20^m$		43 32 21 47 28 23		37 71 67 47 55 57		15 48 21 21 21 26		
46 39 40 28 24 30	Wb 0.971		28 44 51 58 36 27		30 27 57 72 53 60		38 49 23 31 22 17		
41 21 25 23 21 18	22 43 39 33 37 33		30 37 35 30 39 27		44 43 43 56 55 40		26 23 19 15 25 9		
46 35 18 24 33 39	36 39 46 31 41 49		30 25 30 23 40 27		35 42 42 42 42 43		39 26 16 11 16 13		
51 59 13 26 41 39	50 49 33 35 63 30		12 24 28 24 21 27		47 70 37 57 41 32		27 12 12 18 8 20		
51 32 41 43 28 18	39 30 20 36 28 37		21 23 22 24 28 22		32 20 13 31 19 22		17 22 8 23 20 16		
1796 $4^h 40^m$		671 $5^h 20^m$		661 $5^h 40^m$		1041 $6^h 00^m$			
Wj 0.780		Wb 0.858		Wb 0.841		Wd 1.203			
59 60 47 29 28 29	2115 $5^h 00^m$		23 14 24 37 16 15		20 25 5 12 19 16		6 12 15 16 11 10		
83 78 47 31 27 45	S1 1.088		22 24 18 29 20 57		18 9 16 17 25 20		11 24 26 25 27 13		
88 60 32 22 39 31	14 23 33 32 51 41		24 33 37 38 33 37		22 26 33 18 10 18		27 17 19 16 26 20		
175199 59 37 78 42	35 74 44 85 76 54		34 46 29 35 44 29		32 16 29 33 32 24		19 23 17 24 28 25		
73 73 80 49 54 28	54 56 67 63 78 27		25 29 27 38 21 14		20 23 18 11 17 19		16 27 17 30 35 14		
28 25 72 64 47 25	22 44 84 65110 97		19 23 35 30 22 18		23 29 15 16 24 5		10 16 19 7 16 14		
		20 23 53 66 64 40							
		13 23 22 91 33 10							
$\delta = -15^{\circ}$	979 $0^h 00^m$		1081 $0^h 20^m$		1750 $0^h 40^m$		2013 $1^h 00^m$		
	Wd 1.212		Se 0.690		Si 1.064		Wk 0.941		
	24 23 25 23 13 19		61 61 44 48 54 55		45 39 48 61 53 34		30 52 80 83 64 27		
	25 14 35 18 16 40		56 62 46 63 35 33		78 47 52 26 42 32		76 83 66 69 82 70		
	16 15 20 32 18 30		61 47 69 43 33 28		47 57 64 65 53 48		107 61 27 84 66 32		
	18 28 34 39 37 41		53 76 48 65 56 43		42 43 26 27 26 27		88227 54 86 41 48		
	34 27 37 45 42 25		45 75 61 56 42 64		30 48 23 38 26 26		102114 55 75 76 33		
	24 30 30 40 28 27		29 42 36 59 39 32		34 59 41 39 30 12		58 50 46 55 37 19		
	2075 $1^h 20^m$		1008 $2^h 00^m$		2024 $2^h 20^m$		2033 $2^h 40^m$		
	S1 1.333		Sd 1.362		Wk 1.112		Sk 1.266		
41 52 40 34 21 24	1002 $1^h 40^m$		14 17 41 19 16 26		39 30 38 46 36 6		12 21 32 22 18 35		
48 60 55 57 56 49	Wd 1.182		19 60 60 28 13 16		32 60 25 33 27 18		24 13 24 27 31 30		
56 33 21 52 55 64	28 33 31 31 57 45		33 53 47 18 27 28		35 58 17 39 28 38		30 27 20 21 30 38		
56 39 42 43 48 74	22 40 19 27 45 29		40 29 33 46 39 29		36 55 31 39 32 42		20 29 31 31 17 22		
32 41 33 33 62 40	32 45 33 43 77 68		25 30 48 34 23 33		25 57 78 33 37 32		27 28 20 26 19 20		
29 36 31 58 34 44	34 67 48 28 52 71		15 19 25 23 20 15		45 41 42 25 16 14		25 31 46 33 20 45		
737 $3^h 00^m$		2076 $3^h 40^m$		2056 $4^h 00^m$		1009 $4^h 20^m$			
Wb 1.042		W1 0.897		S1 1.358		Sd 1.256			
46 79100 43 37 19	2094 $3^h 20^m$		27 35 33 43 24 26		19 21 22 38 22 22		69 68 23 23 25 10		
35 38 85 41 49 29	S1 1.071		42 69 71 53 50 37		39 47 35 20 32 38		119 40 23 29 57 20		
28 58 56 34 64 31	20 29 25 32 26 59		42 51 59 58 35 36		20 27 39 16 23 29		14 33 29 39 25 24		
54 59 59 40 58 26	26 39 17 33 28 37		43 33 36 46 58 31		27 38 28 16 22 41		19 33 32 40 35 29		
34 38 50 38 37 22	31 32 31 30 24 20		56 56 47 43 61 33		51 26 29 27 25 34		13 18 22 25 34 40		
39 39 27 28 26 25	40 25 39 29 48 52		47 61 34 54 47 31		15 15 20 17 31 27		11 30 21 16 16 20		
1844 $4^h 40^m$		738 $5^h 20^m$		1155 $5^h 40^m$		1124 $6^h 00^m$			
Sj 1.090		Sb 0.985		Se 1.120		Se 0.864			
17 22 36 29 33 77	2296 $5^h 00^m$		14 16 32 33 29 15		11 8 7 12 9 8		2 2 6 4 4 9		
29 38 32 26 45105	S1 0.932		23 28 24 32 25 35		9 9 14 15 21 19		5 9 9 9 5 13		
33 65 39 26 21 19	21 11 36 34 19 9		23 31 20 39 35 32		12 11 15 8 9 17		7 8 11 8 7 12		
37 65 89 22 17 30	43 31 19 43 19 26		11 24 20 42 32 19		13 12 13 16 21 12		11 3 6 13 8 21		
38 68 55 21 15 25	61 33 33 52 35 42		9 24 14 35 40 22		14 12 9 22 16 6		11 18 16 14 20 8		
37 51 41 21 16 20	40 18 23 42 55 30		19 10 24 27 17 12		16 9 4 8 12 14		20 11 19 19 11 17		
		33 41 34 41 47 24							
		22 29 44 32 54 34							

TABLE V. (continued)

		1086	0 ^h 00 ^m	964	0 ^h 20 ^m	1470	0 ^h 40 ^m	2059	1 ^h 00 ^m
		We	0.823	Sd	1.276	Sg	0.885	Sl	1.408
$\delta = -10^\circ$		59 36 57	76 49 53	57 33 27	19 41 34	106 92 73	56102 81	22 25 48	37 33 49
		42 40 47	49 64 39	27 32 34	25 29 16	120 41 66	53 65 50	19 25 30	26 41 68
		26 26 39	50 85 23	33 42 30	27 36 14	88 59 90	146 40 44	27 32 26	35 71 50
		52 44 57	47 78 92	32 34 33	36 36 37	73 47 71	65 40 42	36 37 34	33115 41
		60 38 57	51 56 78	34 40 24	22 40 42	50 64 50	53 62 48	28 31 40	45 50 28
		35 43 38	35 27 40	36 31 20	28 33 33	51 33 52	46 56 35	19 34 54	56 45 18
		1765	1 ^h 20 ^m	1779	1 ^h 40 ^m	2027	2 ^h 20 ^m	2034	2 ^h 40 ^m
		Wi	0.913	Sj	0.787	Sl	1.242	Wl	1.134
40	54 76	73 19 21	100 88 73	80 41 41	47 62 36	80107 59	7 17 19	20 36 36	25 12 20
48	64 88	62 42 27	82 88 78	48 65 56	18 30 39	44 43 49	18 33 52	47 28 14	28 41 59
40	56 64	55 56 36	74 71 60	67111 40	27 34 30	27 52 35	34 35 30	34 31 25	31 40 64
90	48 66	65 48 42	58 53 77	67 70 82	24 48 31	38 34 22	36 38 25	24 35 23	14 39 35
44	55 43	33 63 29	27 41 57	63 47 58	24 44 34	38 18 11	45 33 43	30 28 19	40 34 42
60	61 63	49 42 44	38 61 61	47100 64	16 29 47	26 18 29	40 19 31	25 26 9	18 21 35
		2109	3 ^h 00 ^m	2060	3 ^h 20 ^m	1766	3 ^h 40 ^m	2079	4 ^h 00 ^m
		Sl	1.151	Wl	1.230	Si	1.333	Sl	1.050
23	20 17	18 16 32	32 14 12	21 20 21	17 22 37	16 40 32	38 20 7	10 27 20	30 11 30
34	31 25	23 17 24	10 8 13	26 21 23	53 38 22	12 26 11	60 29 22	29 35 63	28 25 31
25	37 48	26 27 30	32 22 24	14 11 18	27 34 33	18 26 28	30 27 29	28 28 23	28 50 38
29	45 53	52 52 32	30 33 27	32 33 37	18 46 37	18 29 21	29 40 31	29 27 23	48 29 65
33	49 57	50 38 34	21 16 36	20 18 37	28 40 24	21 22 11	16 13 35	44 20 38	43 50 32
36	62 67	23 23 11	15 20 17	32 23 30	11 19 22	33 10 10	20 26 20	31 14 13	75 69 29
		2058	4 ^h 40 ^m	1820	5 ^h 00 ^m	1142	5 ^h 20 ^m	2118	5 ^h 40 ^m
		Wl	0.748	Sj	1.144	Se	0.917	Sl	1.369
23	29 15	55 36 36	16 16 8	38 53 6	10 12 16	12 17 13	3 4 0 0	0 0 6	1 0 0 0
37	37 52	42 36 27	15 20 27	38 28 29	13 19 25	14 8 15	1 0 0 0	2 1 11	2 2 3 1
32	34 47	57 21 37	10 22 25	47 22 18	14 16 30	20 10 13	1 0 0 0	1 5 1	1 3 3 0
27	32 21	36 48 52	12 17 20	45 34 8	11 15 19	21 14 6	3 1 0 0	1 7 5	7 3 2 1
20	19 27	37 49 48	12 11 38	59 30 9	9 7 12	32 17 8	2 1 1 7	5 5 5	8 3 1 2
5	30 46	42 41 84	17 16 27	43 17 6	11 12 31	32 27 14	7 6 3 6	3 3 3	0 3 7 1
		1320	0 ^h 00 ^m	1329	0 ^h 20 ^m	715	0 ^h 40 ^m	1740	1 ^h 00 ^m
		Wl	1.130	Wl	1.006	Sb	1.011	Si	1.149
$\delta = -5^\circ$		29 41 36	39 43 27	94 57 69	54 55 31	74 55 59	70100125	32 45 84	80 69 59
		40 44 38	48 54 49	77 60 62	58 65 45	106 57 47	37100 62	33 53 61	122110 90
		28 28 26	44 34 30	49 32 46	46 26 21	61 92 52	47 59 43	33 33 75	43 46 55
		43 26 21	36 25 36	53 49 29	19 32 37	52 49 41	45 52 70	31 30 49	51 36 37
		61 32 33	22 44 18	60 39 28	39 40 45	50 62 39	28 47 47	47 40 57	32 65 28
		44 27 33	53 38 31	46 35 27	25 34 42	72 61 54	44 56 62	18 24 46	35 42 47
		998	1 ^h 20 ^m	2016	1 ^h 40 ^m	1342	2 ^h 00 ^m	1827**	2 ^h 20 ^m
		Sd	1.351	Wk	1.117	Sg	1.008	Wj	0.484
60	51 41	45 35 32	39 58 50	55 28 54	50 28 39	43 76 51	54 95 67	76 95 75	14 17 21
47	45 39	35 43 23	77 60 51	62 42 43	43 30 32	44 59 77	60 86 61	48 75 54	18 24 26
41	49 49	23 10 13	64101 62	58 60 60	26 52 33	34 54 56	44 66 74	73 56 29	15 28 37
23	39 40	21 23 20	50 39 49	85 87 36	47 39 44	37 67 46	95 75110 63	51 46	24 11 36
27	33 36	20 23 41	56 50 51	75 44 22	65 60 63	45 89 78	24 47 87	67 62 73	23 15 27
21	44 52	57 10 14	82 73 60	47 40 36	58 58 46	83135 64	16 24 40	35 51 54	22 14 18
		2128	3 ^h 00 ^m	688	3 ^h 20 ^m	999	3 ^h 40 ^m	2083	4 ^h 00 ^m
		Sl	1.171	Sb	0.856	Wd	1.222	Sl	1.408
64	55 27	18 26 20	43 80 56	55 50 86	14 20 19	49 18 16	10 12 20	13 8 14	18 16 32
29	55 20	31 20 13	55 96 54	32 34 32	29 17 25	29 46 31	24 13 18	19 12 23	26 36 54
26	59 18	28 24 13	72 56 46	40 28 31	24 32 74	73 53 46	34 19 25	9 20 20	53 50 36
26	22 29	22 32 20	49 40 35	32 15 32	31 23 17	45 48 28	32 18 14	24 43 16	52 43 33
23	25 35	22 20 24	41 27 40	31 21 25	25 17 28	42 32 20	43 22 15	19 24 16	18 53 45
23	27 17	15 18 19	49 15 17	21 27 14	22 25 40	20 31 28	27 21 9	2 14 11	36 15 39
		657	4 ^h 40 ^m	1150*	5 ^h 00 ^m	2110	5 ^h 20 ^m	2120*	5 ^h 40 ^m
		Sb	1.260	Se	0.800	Sl	1.176	Wl	1.105
19	36 30	46 56 14	16 28 18	39 12 13	1 0 5 5	4 11 11	1 0 1 2	8 2 2	0 1 0 1
34	34 35	40 29 14	23 21 24	66 19 48	4 2 14 8	16 16 16	2 5 0 3	6 3 6 3	1 3 1 0
38	18 22	12 27 39	10 29 14	26 38 31	1 9 6 13	10 7 7	0 0 0 1	0 1 0 1	1 0 0 1
28	41 21	11 26 48	18 15 30	40 42 35	1 16 14	8 7 12	4 1 1 0	0 1 0 1	1 0 1 1
27	26 25	8 20 14	8 17 22	31 49 38	4 19 19	14 11 5	1 2 0 0	1 3 0 0	0 1 0 0
5	8 4	30 16 26	18 25 13	52 64 11	8 6 6 11	12 10 10	6 3 2 1	0 6 0 6	1 2 0 1
		2111	6 ^h 00 ^m	2111	6 ^h 00 ^m	2111	6 ^h 00 ^m	2111	6 ^h 00 ^m
		Sl	1.230	Sl	1.230	Sl	1.230	Sl	1.230
0	1 0 1	1 1 1 1	0 1 0 1	1 1 1 1	0 1 0 1	1 1 1 1	0 1 0 1	1 1 1 1	0 1 0 1
1	3 1 0	2 0 2 0	1 3 1 0	2 0 2 0	1 3 1 0	2 0 2 0	1 3 1 0	2 0 2 0	1 3 1 0
2	5 3 0	4 2 0 0	2 5 3 0	4 2 0 0	2 5 3 0	4 2 0 0	2 5 3 0	4 2 0 0	2 5 3 0
3	7 5 0	6 4 0 0	3 7 5 0	6 4 0 0	3 7 5 0	6 4 0 0	3 7 5 0	6 4 0 0	3 7 5 0
4	9 7 0	8 6 0 0	5 9 7 0	8 6 0 0	5 9 7 0	8 6 0 0	5 9 7 0	8 6 0 0	5 9 7 0
5	11 9 0	10 8 0 0	7 11 9 0	10 8 0 0	7 11 9 0	10 8 0 0	7 11 9 0	10 8 0 0	7 11 9 0

TABLE V. (continued)

		981	0 ^h 00 ^m	1729	0 ^h 20 ^m	675	0 ^h 40 ^m	1744	1 ^h 00 ^m
		Wd	1.967	Si	1.329	Sb	0.987	Si	1.033
$\delta = 0^\circ$		24 25 16 23 22 22		58 44 26 14 37 26		42 47 50 71 75 77		54 67 54 33 23 38	
		28 21 32 20 30 22		77 56 37 51 55 29		48 70 63 48 90 86		67 98 50 27 42 55	
		40 30 54 38 36 22		49 41 37 31 49 51		77 75 85 81 43 70		81 65 66 48 90 57	
		42 42 23 22 23 13		45 89 44 39 76 51		59 49 75 67 55 56		89 95 75 58 87 41	
		31 42 20 20 63 14		41 56 85 107 51 37		59 76 90 62 52 51		38 76 114 44 202 50	
		18 34 38 30 33 26		81 33 53 36 32 15		70 49 60 70 108 111		34 46 103 74 63 61	
2073	1 ^h 20 ^m	1761	1 ^h 40 ^m	655	2 ^h 00 ^m	982	2 ^h 20 ^m	2114	2 ^h 40 ^m
Wl	1.075	Si	1.097	Wb	0.768	Sd	1.852	Wl	0.862
1 28 42 58 51 38		33 40 45 45 42 27		37 67 41 47 49 30		17 8 4 12 8 9		26 31 38 39 35 40	
8 43 133 85 71 52		62 41 32 56 50 27		35 65 57 39 50 54		40 13 4 13 17 22		37 35 48 62 95 53	
7 34 73 64 101 66		65 53 37 41 38 59		49 65 92 68 67 74		33 16 13 10 8 11		37 46 52 43 35 55	
9 55 73 122 123 89		51 55 44 49 84 39		51 66 73 50 81 55		19 20 12 7 13 11		40 80 71 24 53 47	
0 78 112 67 56 40		56 39 39 45 37 47		44 73 88 128 67 52		25 31 25 6 12 14		53 48 32 41 45 42	
2 65 56 46 56 37		44 58 43 56 28 53		58 43 66 61 79 45		6 31 21 15 19 23		27 34 44 35 30 17	
2277	3 ^h 00 ^m	678	3 ^h 20 ^m	2074	3 ^h 40 ^m	1762	4 ^h 00 ^m	2298	4 ^h 20 ^m
Sm	1.288	Wb	0.860	Sl	1.268	Si	1.248	Sm	0.772
2 23 33 31 33 22		46 34 48 29 40 62		9 11 23 19 19 27		24 13 2 3 6 4		22 22 25 44 43 42	
3 27 30 28 29 19		35 24 31 51 44 30		21 19 19 14 24 21		19 19 8 10 14 12		39 30 18 28 49 29	
7 41 50 36 31 36		65 33 42 59 31 41		18 17 22 15 31 39		10 11 6 12 11 11		43 37 26 30 45 10	
5 45 39 19 28 30		45 66 86 32 48 70		21 21 25 18 20 20		11 30 12 12 10 14		39 38 46 40 49 24	
9 44 46 30 29 33		63 61 46 57 48 60		16 18 16 20 21 32		14 37 20 8 4 12		23 44 32 35 45 15	
0 49 21 18 24 19		36 60 56 55 57 61		14 24 19 60 21 17		20 18 27 17 14 16		28 22 43 63 27 23	
1828	4 ^h 40 ^m	1146	5 ^h 00 ^m	1509	5 ^h 20 ^m	2095	5 ^h 40 ^m	2305	6 ^h 00 ^m
Wj	0.804	Se	1.075	Sh	0.875	Sl	1.230	Wm	1.084
8 19 18 11 13 12		23 13 30 43 27 27		7 6 14 11 12 25		0 0 2 3 10 6		2 0 0 0 0 0	
0 28 32 22 15 40		26 8 29 16 42 35		19 12 16 34 30 28		0 0 4 1 9 12		0 0 0 0 1 1	
9 57 39 60 37 38		25 24 27 24 17 20		7 14 18 21 20 27		0 0 7 3 9 0		0 1 0 0 0 0	
1 37 35 33 38 29		37 43 33 33 36 9		9 14 18 20 24 43		0 0 1 5 5 4		0 0 1 0 0 2	
0 37 28 40 32 17		16 24 34 23 28 22		5 16 11 15 15 23		0 0 0 1 4 0		0 0 2 2 0 0	
6 40 52 53 60 25		13 19 19 29 5 17		4 6 9 13 16 15		0 0 0 0 5 0		0 0 1 0 0 0	
$\delta = +5^\circ$		1726	0 ^h 00 ^m	1028	0 ^h 20 ^m	1033	0 ^h 40 ^m	677	1 ^h 00 ^m
		Wl	0.882	Sd	1.171	Sd	1.059	Wb	1.096
		21 31 46 68 33 45		43 38 42 47 34 20		33 51 36 41 38 52		44 47 47 37 53 29	
		42 62 57 55 38 71		43 46 50 74 83 31		16 35 38 101 66 43		38 34 63 44 30 12	
		54 58 74 79 74 74		66 41 44 41 46 33		43 31 48 37 59 48		40 55 45 35 50 51	
		56 77 92 49 41 34		69 33 36 48 53 46		32 28 45 43 45 62		36 49 73 44 62 19	
		58 79 72 73 71 59		42 49 31 46 20 42		39 28 36 71 67 49		38 44 65 21 32 28	
		49 56 48 42 42 51		56 48 32 18 39 23		38 45 48 63 51 60		57 70 56 31 33 26	
985	1 ^h 20 ^m	2019	1 ^h 40 ^m	2062	2 ^h 00 ^m	978	2 ^h 20 ^m	1029	2 ^h 40 ^m
Sd	1.292	Wk	1.001	Sl	1.290	Wd	1.428	Sd	0.994
7 20 48 37 27 29		41 29 46 54 38 29		37 29 39 35 28 22		12 21 13 14 25 23		36 27 55 57 47 24	
7 34 45 44 35 20		44 29 59 68 48 61		27 28 20 41 37 40		34 27 22 20 30 11		33 23 29 49 39 52	
6 30 23 44 34 23		45 26 40 60 52 38		21 42 15 30 51 32		23 7 20 14 21 21		48 71 45 25 30 38	
2 52 30 56 37 23		34 32 23 29 59 50		39 42 45 28 28 32		30 23 22 23 22 26		37 29 44 28 25 33	
2 24 34 46 58 33		27 46 38 45 40 40		32 34 32 38 42 18		45 30 19 10 15 21		32 49 40 31 21 49	
2 29 38 47 39 33		36 38 58 49 35 31		19 36 30 28 30 17		23 13 10 16 15 12		30 27 29 38 34 39	
2104	3 ^h 00 ^m	2136	3 ^h 20 ^m	1818	3 ^h 40 ^m	1822	4 ^h 00 ^m	2063	4 ^h 20 ^m
Wl	1.105	Sl	1.335	Wj	0.540	Sj	1.109	Wl	1.068
11 5 14 26 28 25		11 29 36 22 8 15		18 37 21 26 10 13		9 11 12 17 21 14		14 24 16 16 10 9	
11 21 27 45 65 23		18 15 21 12 9 17		40 43 36 35 25 41		20 20 21 28 15 16		21 25 30 19 16 19	
3 35 37 82 112 40		14 18 13 14 18 19		34 38 33 42 21 34		21 11 23 31 11 13		8 17 15 15 12 13	
2 36 34 38 40 25		17 28 19 18 28 13		26 31 32 48 38 41		13 6 19 8 15 7		3 10 15 9 24 5	
0 60 71 69 45 24		34 27 27 23 31 20		28 27 27 62 73 56		8 18 14 12 8 5		12 28 19 33 22 9	
1 37 57 45 46 23		35 32 34 20 27 38		24 40 43 49 63 56		27 11 7 5 9 6		19 23 18 33 26 25	
2102	4 ^h 40 ^m	1042	5 ^h 00 ^m	2129	5 ^h 20 ^m	1517	5 ^h 40 ^m	2137	6 ^h 00 ^m
Sl	1.034	Wd	1.382	Sl	1.018	Wh	1.032	Sl	1.230
15 29 28 16 14 15		17 14 21 35 25 12		6 6 4 1 8 22		4 1 1 0 3 4		2 0 4 1 0 4	
36 32 23 25 18 19		16 21 16 26 21 20		10 7 3 25 116 19		3 4 1 5 5 10		1 1 1 1 1 3	
40 45 36 24 8 12		12 26 20 30 14 17		6 20 10 29 39 10		2 0 2 2 5 6		0 1 1 0 1 1	
58 20 28 15 4 5		7 12 29 23 11 29		8 13 9 24 43 6		0 2 2 0 4 7		1 0 0 1 0 1	
25 14 13 3 1 4		9 21 24 23 14 13		7 16 15 40 10 16		1 1 0 4 2 4		1 0 0 1 2 0	
27 11 8 7 5 9		17 12 17 21 20 18		4 8 11 12 7 18		0 0 1 2 13 5		1 2 1 0 1 0	

TABLE V. (continued)

		1780		1809		1332		703	
		Wi	0 ^h 00 ^m 1.130	Sj	0 ^h 20 ^m 1.077	Si	0 ^h 40 ^m 1.136	Sb	1 ^h 00 ^m 0.915
$\delta = +10^\circ$		28 45 45	60 49 25	40 31 36	29 32 16	14 17 9	26 39 37	51 77 29	74 54 13
		29 40 30	27 61 50	41 101 42	33 41 17	44 37 19	38 30 32	56 31 40	41 42 46
		49 38 48	27 29 59	37 33 40	44 30 25	38 38 48	15 33 40	58 55 39	22 49 39
		34 29 27	13 31 45	28 41 40	33 18 21	30 28 24	21 19 25	99 45 46	33 31 33
		30 31 51	51 42 37	36 25 61	47 46 23	57 27 43	36 33 38	46 60 42	41 52 56
		35 30 45	56 32 38	51 39 45	40 28 8	28 45 27	37 27 47	52 48 76	58 58 27
		1339	1 ^h 20 ^m Wi 0.912	2055	1 ^h 40 ^m Si 1.474	692	2 ^h 00 ^m Wb 0.678	980	2 ^h 20 ^m Sd 1.476
31 46 32		63 63 34	30 59 27	22 39 24	33 35 33	45 56 48	14 28 19	6 16 13	70 60 51
44 64 64		77 51 57	16 31 29	15 29 14	45 48 38	36 39 30	23 17 19	17 14 20	71 51 38
17 54 66		60 40 48	19 38 44	16 17 16	58 80 76	71 51 30	26 17 31	20 27 17	50 41 59
25 44 78		45 49 55	22 24 18	20 8 12	63 79 69	81 54 63	57 36 22	26 23 20	45 43 42
25 44 124		48 37 43	42 31 31	18 11 16	62 56 68	81 72 89	22 20 24	17 11 35	36 42 29
29 36 60		52 52 41	20 20 34	54 27 12	68 73 73	66 54 54	23 24 21	16 27 32	37 34 44
		1751	3 ^h 00 ^m Si 0.895	2280	3 ^h 20 ^m Wm 0.890	1772	3 ^h 40 ^m Si 1.517	1813	4 ^h 00 ^m Sj 0.928
11 19 24		30 123 48	15 12 9	12 6 11	7 7 12	10 6 7	3 4 7	9 17 10	3 3 8
37 27 20		37 46 58	23 13 13	7 6 14	16 17 7	10 5 7	2 4 10	14 6 14	6 6 6
22 31 44		32 54 42	18 22 12	6 8 8	16 16 10	24 18 10	2 8 7	13 19 16	5 11 5
52 48 54		31 44 29	23 25 41	12 22 26	8 16 8	30 26 8	7 12 9	18 14 12	7 2 8
8 11 29		27 35 27	33 49 21	18 19 5	8 17 11	11 14 11	9 15 17	15 9 8	14 9 8
27 7 14		22 34 23	10 42 45	33 17 20	9 18 8	13 8 3	8 17 13	15 21 13	11 18 16
		766	4 ^h 40 ^m Sb 0.926	707	5 ^h 00 ^m Sb 1.005	2105	5 ^h 20 ^m Wl 1.226	689	5 ^h 40 ^m Sb 1.000
9 3 5		5 5 3	2 2 1	7 9 6	0 0 0	0 4 1	2 1 1	4 4 1	0 1 1
3 20 10		5 5 8	3 7 14	7 2 3	0 3 0	0 0 0	1 0 2	1 5 1	0 0 0
18 6 6		13 11 10	6 14 10	13 9 16	0 1 2	2 0 6	2 1 2	5 5 0	1 0 1
17 33 14		30 30 5	15 20 19	28 19 7	1 2 3	5 5 7	3 5 2	3 5 3	0 0 4
15 43 45		12 27 15	10 16 31	24 13 10	3 9 3	4 8 6	1 1 0	2 7 13	0 0 2
11 23 29		17 14 15	27 18 29	35 20 9	4 4 2	1 12 21	5 3 2	1 2 5	2 0 1
		944	0 ^h 00 ^m Wc 1.101	2032	0 ^h 20 ^m Wk 1.304	1737	0 ^h 40 ^m Si 1.218	687	1 ^h 00 ^m Wb 0.792
$\delta = +15^\circ$		43 38 45	43 34 62	21 33 21	31 48 28	28 29 34	26 24 31	115 99 36	45 47 42
		32 37 48	40 47 72	36 42 22	36 38 30	30 39 34	46 28 25	149 95 44	57 47 35
		55 49 73	60 62 63	13 28 27	34 33 17	59 53 42	37 24 26	192 68 66	55 51 71
		50 58 36	63 57 40	30 20 30	20 44 35	27 21 42	35 30 27	109 109 89	86 79 47
		39 49 60	57 68 59	14 27 50	19 45 17	19 13 19	40 28 18	77 147 76	75 57 34
		28 46 38	53 52 24	27 22 33	18 25 21	17 20 11	22 38 32	72 100 63	89 58 17
		1336	1 ^h 20 ^m Si 1.190	2091	1 ^h 40 ^m Wl 1.187	1077	2 ^h 00 ^m Sc 1.114	1791	2 ^h 20 ^m Sj 0.876
37 24 35		39 44 101	42 29 48	32 44 37	33 41 33	50 33 35	17 9 21	34 39 47	25 33 34
41 42 27		26 81 121	50 46 51	21 39 37	42 49 28	22 40 39	6 18 33	32 29 52	26 37 50
52 47 22		31 66 110	41 30 28	41 63 27	43 62 67	44 27 40	23 25 23	48 36 44	41 56 51
33 39 38		70 53 68	47 43 26	30 41 31	48 57 56	41 56 48	25 7 21	19 50 41	37 40 44
37 28 42		32 60 41	53 109 61	47 33 43	43 28 42	25 38 33	16 28 27	32 32 49	64 46 31
32 36 18		59 39 25	39 65 30	33 44 36	31 33 25	31 40 37	28 46 28	19 25 32	29 33 18
		2358	3 ^h 00 ^m Wm 1.154	2352	3 ^h 20 ^m Sm 1.123	1769	3 ^h 40 ^m Si 1.012	1074	4 ^h 00 ^m Sc 1.181
22 23 22		18 12 18	12 19 19	22 24 24	11 16 8	11 9 10	4 3 6	3 6 12	10 11 10
52 32 28		33 30 17	12 19 33	17 37 56	16 23 15	20 11 12	10 3 5	6 15 10	15 12 10
39 42 61		33 121 54	29 18 36	19 40 34	16 11 16	74 32 23	7 10 9	11 11 14	6 6 8
32 30 39		46 27 33	29 18 23	24 23 25	15 9 6	30 14 18	0 6 12	12 13 12	13 10 20
12 19 19		105 71 65	16 17 11	15 12 5	21 30 6	21 10 9	14 8 29	16 9 12	19 19 23
13 11 25		29 101 45	12 10 5	9 7 6	13 10 16	12 9 14	9 6 6	6 15 6	10 13 19
		1123	4 ^h 40 ^m Se 1.056	1811	5 ^h 00 ^m Wj 1.095	1841	5 ^h 20 ^m Sj 0.733	1154	5 ^h 40 ^m We 0.813
1 2 0		3 5 5	7 14 3	1 2 4	5 5 9	12 11 6	2 9 9	1 2 4	2 0 0
1 2 3		0 5 11	5 9 9	2 4 5	6 5 9	9 6 8	1 2 11	9 6 8	0 1 0
5 2 3		4 3 2	2 5 1	8 4 3	4 1 9	8 12 6	3 4 5	9 6 1	0 0 0
2 5 1		3 1 3	2 1 6	2 2 1	4 7 9	8 3 4	4 4 6	4 5 0	1 0 0
5 4 3		2 11 10	6 3 3	6 9 10	9 3 4	8 6 8	0 3 3	2 8 6	0 0 0
6 2 2		5 3 2	5 6 2	8 7 7	1 4 4	4 7 4	4 2 2	5 3 2	0 0 0

TABLE VI. AREA II, UNCORRECTED COUNTS OF NEBULAE PER SQUARE DEGREE

		1041	6 ^h 00 ^m	1160	6 ^h 20 ^m	713	6 ^h 40 ^m	1050	7 ^h 00 ^m
		Wd	0.953	Se	1.090	Sb	0.940	Sd	1.172
$\delta = -20^\circ$		6	12 15 16 11 10	3	4 4 1 6 7	0	1 1 0 1 2	0	0 0 0 0 0 1
		11	24 26 25 27 13	0	3 2 9 5 10	0	0 1 3 0 5	0	0 0 0 0 0 0
		27	17 19 16 26 20	4	7 5 3 8 15	1	4 0 3 2 3	0	1 0 2 1 2
		19	23 17 24 28 25	5	4 7 23 12 10	6	0 0 1 6 3	0	0 0 3 3 3
		16	27 17 30 35 14	7	13 14 6 19 16	1	2 1 4 3 3	0	0 0 1 1 2
		10	16 19 7 16 14	5	8 18 21 15 6	0	0 2 4 2 6	0	0 1 0 1 0
		1151	7 ^h 20 ^m	739	7 ^h 40 ^m	1449	8 ^h 00 ^m	1125	8 ^h 20 ^m
		We	0.789	Sb	0.940	Sg	1.022	Se	0.866
0 0 0 0 0 0		0	0 2 0 0 0 0	5	4 1 0 2 1	7	11 9 3 6 3	8	1 14 4 2 5
0 0 1 2 1 0		0	1 0 0 0 1	6	17 5 2 1 0	22	5 11 5 9 7	0	6 7 7 12 17
0 0 0 1 0 0		0	0 1 0 0 0 0	2	4 4 3 1 0	14	13 4 8 3 2	5	20 9 11 13 8
0 1 0 1 0 1		1	0 1 0 0 0 0	10	7 1 4 1 0	11	10 14 10 9 12	8	12 14 20 13 9
0 0 0 0 1 0		1	0 0 1 0 0 0	3	3 2 0 0 1	17	14 11 10 10 2	7	8 7 8 5 7
0 0 1 1 2 1		0	0 1 0 0 2	1	6 10 1 0 1	9	5 7 10 6 3	8	1 5 1 4 8
		714	9 ^h 00 ^m	1846	9 ^h 20 ^m	787	9 ^h 40 ^m	740	10 ^h 00 ^m
		Sb	0.898	Sj	0.983	Sb	0.984	Sb	0.963
11 9 9 8 2 12		19	13 23 15 18 12	12	21 25 21 20 14	18	9 11 24 8 6	15	16 26 33 43 17
14 15 18 10 11 3		16	13 14 11 24 5	23	30 19 30 13 15	26	20 21 41 29 22	24	32 30 29 25 34
10 6 18 9 7 7		12	19 14 11 13 10	10	13 23 39 24 8	27	20 27 51 18 13	16	21 48 32 29 31
15 7 11 3 9 8		16	17 15 17 8 5	8	14 21 50 46 12	20	27 18 27 22 12	24	25 19 7 28 15
7 4 5 5 0 7		19	13 20 9 4 4	9	18 20 15 31 20	15	12 31 28 24 16	13	20 15 28 25 13
2 10 5 8 2 5		7	15 10 8 5 2	8	10 20 17 12 8	11	16 16 17 21 2	10	9 11 6 9 14
		773	10 ^h 40 ^m	1523	11 ^h 00 ^m	1180	11 ^h 20 ^m	1586	11 ^h 40 ^m
		Sb	0.966	Sh	0.652	Se	1.119	Sh	0.869
25 36 49 29 26 16		33	39 34 35 49 22	16	11 8 15 29 16	17	40 39 51 33 15	35	36 38 27 22 36
22 33 45 29 34 18		31	22 21 61 38 36	13	13 11 16 16 10	36	38 31 17 38 17	68	69 37 49 73 40
48 28 47 28 16 18		41	30 29 20 34 33	15	16 23 14 15 17	30	28 14 31 20 18	51	44 34 49 51 36
22 35 37 33 38 21		36	45 29 34 38 27	13	18 8 14 35 10	41	46 16 30 28 21	31	39 54 67 41 62
19 27 24 18 23 19		41	25 69 23 27 16	15	17 12 14 24 27	29	47 26 21 21 24	21	34 36 53 30 15
18 20 24 23 12 15		22	21 22 25 14 16	11	21 18 14 18 10	9	9 17 15 20 10	29	13 23 28 14 10
		1124	6 ^h 00 ^m	1521	6 ^h 20 ^m	778	6 ^h 40 ^m	1845	7 ^h 00 ^m
		Se	0.742	Sh	1.149	Sb	0.940	Wj	0.766
$\delta = -15^\circ$		2	2 6 4 4 9	0	4 1 2 6 3	0	0 1 1 0 0	1	0 0 0 0 0 0
		5	9 9 9 5 13	2	2 2 6 6 4	0	0 1 1 1 5	0	0 0 0 0 0 1
		7	8 11 8 7 12	3	0 4 5 2 9	0	0 0 0 3 1	0	0 1 1 1 0
		11	3 6 13 8 21	0	3 6 6 4 4	0	1 0 1 1 0	1	0 0 0 1 1 0
		11	18 16 14 20 8	2	6 8 5 3 6	0	3 0 1 3 4	0	1 0 1 1 1
		20	11 19 19 11 17	2	6 2 2 10 3	1	2 1 0 1 2	0	0 0 1 0 0 0
		2375	7 ^h 20 ^m	2500	7 ^h 40 ^m	1156	8 ^h 00 ^m	1158	8 ^h 20 ^m
		Sm	0.979	Wn	1.120	Se	1.262	Se	0.865
0 0 1 0 0 0		6	0 0 1 0 0 0	5	4 3 6 1 0	10	11 12 15 5 11	10	13 18 9 11 11
0 0 1 0 1 0		0	5 1 0 0 1	2	13 3 1 2 1	20	18 16 21 9 4	11	13 17 16 24 15
0 0 0 0 0 0		1	7 0 2 0 0	4	2 2 6 2 2	7	23 25 18 9 4	26	25 16 25 19 10
3 1 2 1 0 2		2	3 0 2 2 0	8	6 1 2 0 1	8	11 16 9 11 8	13	10 26 14 17 7
2 1 1 1 0 1		0	1 0 0 0 1	6	4 3 2 0 0	3	10 3 3 9 7	13	9 14 14 11 6
0 0 2 1 1 0		0	0 1 1 0 0	4	2 1 0 1	9	11 13 4 5 2	10	9 15 3 2 5
		1179	9 ^h 00 ^m	1877	9 ^h 20 ^m	1883	9 ^h 40 ^m	801	10 ^h 00 ^m
		Se	1.054	Sj	0.904	Wj	0.704	Sb	1.021
16 11 27 17 13 12		25	34 21 41 33 24	50	46 44 34 34 33	20	39 24 22 29 45	46	32 24 49 38 34
6 20 12 13 13 7		15	17 25 16 10 14	39	63 30 36 25 16	12	25 20 49 26	42	23 34 41 27 35
25 20 14 19 9 18		31	23 14 5 15 26	55	50 43 38 33 32	30	46 33 43 39 35	32	39 41 35 16 44
31 19 21 24 13 10		33	22 16 8 34 31	67	41 31 26 29 24	30	43 62 41 58 60	43	31 53 43 29 36
24 13 17 18 15 9		26	27 10 18 19 19	29	42 38 22 34 29	43	32 19 16 30 14	37	30 41 23 38 46
15 10 13 11 3 9		11	12 18 17 26 13	14	35 29 28 29 18	21	9 11 22 8 10	28	23 37 37 55 24
		1488	10 ^h 40 ^m	2366	11 ^h 00 ^m	2563	11 ^h 20 ^m	1878	11 ^h 40 ^m
		Sg	0.714	Wn	1.454	So	1.245	Sj	1.140
45 54 68 33 41 44		19	30 14 28 22 20	40	33 23 32 33 19	43	27 49 68 41 34	45	39 49 42 54 52
50 55 49 50 44 29		28	27 25 34 24 19	70	33 40 18 19 31	28	19 39 46 94 47	63	66 54 44 45 39
55 55 48 56 44 33		30	47 47 30 24 25	50	38 34 41 21 36	45	33 39 41 25 48	49	45 41 37 77 57
45 47 56 39 30 31		22	28 58 27 20 14	27	22 10 10 19 17	58	58 41 17 44 29	49	39 29 23 49 76
53 80 54 53 31 28		20	12 28 37 24 29	15	22 14 10 6 7	36	48 49 47 26 25	51	52 29 38 48 65
40 45 65 38 30 22		20	24 20 15 31 7	9	5 7 10 14 7	19	43 38 42 27 12	47	36 50 39 28 32

TABLE VI. (continued)

$\delta = -10^\circ$		1157 S _m	6 ^h 00 ^m 0.944	2080 S _l	6 ^h 20 ^m 1.127	2364 S _m	6 ^h 40 ^m 0.979	2103 S _l	7 ^h 00 ^m 1.127
		1 0 0	0 3 3	2 1 0	0 1 1	0 2 0	1 0 1	0 0 0	0 0 0
		2 2 3	1 3 0	0 0 0	3 1 0	1 1 2	1 1 0	0 0 0	0 0 0
		1 3 3	0 2 0	0 0 0	0 0 4	0 0 1	1 0 0	0 0 0	0 1 1
		7 3 2	1 0 1	0 0 1	2 0 7	0 0 1	0 1 1	0 0 0	0 0 3
		8 3 1	2 2 2	0 0 1	1 1 4	0 0 0	2 1 1	1 0 0	0 1 0
		0 3 7	1 1 6	2 1 0	2 6 1	0 0 2	0 0 1	0 0 0	0 0 0
		1147 We	7 ^h 20 ^m 0.789	1134 Se	7 ^h 40 ^m 0.944	1852 Wj	8 ^h 00 ^m 0.844	1461 Sg	8 ^h 20 ^m 0.966
		2 2 0	1 1 2	3 3 3	8 1 1	25 8 9	11 7 3	6 11 22	6 17 12
		1 1 1	0 0 3	5 6 3	1 2 0	2 8 12	15 13 7	19 29 26	22 14 2
		1 0 2	1 0 1	2 1 0	3 0 0	5 0 30	11 3 2	14 20 21	20 14 3
		1 0 0	0 0 1	1 1 3	0 1 1	25 17 13	9 5 0	24 15 15	23 11 12
		1 0 0	0 1 2	2 1 3	1 0 1	13 7 11	5 5 2	12 7 17	14 7 14
		0 0 0	0 0 0	0 0 2	1 0 1	10 6 9	2 2 1	11 15 13	18 6 7
		1467 Sg	9 ^h 00 ^m 0.779	2398 Wn	9 ^h 20 ^m 1.062	1884 Sj	9 ^h 40 ^m 1.203	2501 Wn	10 ^h 00 ^m 0.883
		28 20 26	15 33 23	16 25 22	32 39 23	14 16 8	22 9 14	49 31 34	49 22 23
		35 33 45	22 35 19	14 16 27	43 40 27	13 19 14	21 30 21	29 31 30	41 20 25
		56 159 51	29 32 17	21 19 27	40 26 30	19 22 23	26 30 21	61 43 51	20 42 21
		52 36 33	32 19 20	16 12 25	22 34 19	20 41 18	25 29 11	15 18 22	25 25 19
		29 20 20	21 17 16	20 15 20	23 25 15	20 40 22	29 27 26	24 38 38	22 22 27
		29 19 50	21 16 17	20 29 35	30 30 15	26 39 41	25 26 19	18 36 31	25 35 34
		452 Wa	10 ^h 40 ^m 0.579	1210 Se	11 ^h 00 ^m 0.967	2844 So	11 ^h 20 ^m 1.428	2551 So	11 ^h 40 ^m 1.226
		78 37 67	51 38 35	60 39 30	35 53 52	35 29 45	32 32 27	29 24 41	30 32 35
		115 102 120	134 53 55	29 24 39	26 41 45	39 33 28	18 17 16	23 22 29	55 51 60
		84 132 65	68 86 60	19 37 42	44 33 38	64 37 27	23 14 20	19 37 41	55 72 57
		107 60 71	89 44 63	33 39 21	33 43 46	50 41 29	38 27 20	34 44 35	79 63 43
		49 50 59	75 72 43	24 31 33	62 28 21	41 34 20	20 19 19	30 57 77	82 54 46
		40 53 73	35 42 62	28 40 23	36 42 26	23 31 23	28 21 14	40 39 58	53 35 31
		2111 S _l	6 ^h 00 ^m 1.127	2084 S _l	6 ^h 20 ^m 1.127	2141 S _l	6 ^h 40 ^m 1.127	2112 W _l	7 ^h 00 ^m 0.942
$\delta = -5^\circ$		0 1 0	1 1 1	0 0 0	0 0 0	0 0 0	1 0 0	1 1 0	0 0 0
		1 3 1	0 2 0	0 0 2	0 0 0	0 0 0	0 0 1	3 1 0	3 0 0
		1 0 0	1 1 0	0 1 0	1 1 1	0 0 0	0 0 0	0 0 0	0 2 0
		1 0 1	1 1 0	0 0 0	3 0 1	0 0 0	0 0 0	0 0 0	0 0 0
		0 1 0	0 0 0	0 0 2	0 0 0	0 0 0	0 0 0	0 0 0	0 0 1
		1 2 0	1 2 2	1 0 0	0 0 0	0 1 0	0 2 1	0 0 0	0 0 1
		786 S _b	7 ^h 20 ^m 0.940	1143 Se	7 ^h 40 ^m 1.329	2119 S _l	8 ^h 00 ^m 1.012	1487 Wg	8 ^h 20 ^m 0.659
		3 1 2	2 1 0	6 8 11	3 3 1	5 14 16	12 14 10	27 41 32	33 27 11
		1 2 0	0 0 1	17 12 4	4 4 1	10 10 18	16 20 17	49 22 23	38 12 11
		1 3 3	0 1 0	14 2 1	1 0 1	11 9 8	19 27 20	45 14 28	23 17 10
		1 3 0	1 0 0	5 4 1	0 0 1	8 8 13	25 16 6	46 27 33	37 26 8
		2 1 2	1 0 0	7 4 1	1 1 1	8 21 12	12 10 6	20 17 26	17 22 16
		0 2 1	2 0 0	2 2 1	2 1 0	9 13 9	6 4 5	13 23 25	10 19 19
		1833 Sj	9 ^h 00 ^m 0.631	1201 Se	9 ^h 20 ^m 1.297	1148 We	9 ^h 40 ^m 0.583	435 Sa	10 ^h 00 ^m 1.106
		30 31 37	27 35 23	39 21 25	21 22 16	42 54 64	40 63 59	59 41 35	66 39 27
		25 36 26	36 34 20	23 16 12	32 25 9	43 60 62	70 65 37	55 40 42	56 45 31
		30 20 31	58 44 37	28 21 8	30 24 14	69 76 91	83 99 44	27 54 62	44 40 42
		28 34 35	54 64 30	29 13 16	18 19 12	59 54 52	62 56 42	44 126 58	34 33 29
		21 27 31	41 22 39	22 26 27	22 19 9	60 41 52	40 40 34	27 61 57	39 47 27
		29 27 25	21 31 33	16 26 18	24 34 11	42 34 31	35 20 23	44 35 36	40 20 14
		443 Sa	10 ^h 40 ^m 1.221	2144 Wl	11 ^h 00 ^m 1.130	1834 Sj	11 ^h 20 ^m 0.831	1202 We	11 ^h 40 ^m 0.738
		23 41 37	34 24 45	35 52 33	33 32 21	43 28 34	61 34 59	120 90 90	85 74 48
		37 62 35	52 37 40	36 43 50	32 25 26	99 46 61	57 70 58	99 114 105	104 80 94
		34 52 34	14 15 18	44 59 47	38 38 24	95 78 66	73 67 54	69 65 57	60 68 87
		47 36 17	24 20 21	44 64 49	40 55 37	77 92 43	87 80 49	72 73 79	47 51 68
		35 17 20	11 36 14	76 52 72	89 59 39	48 56 31	34 49 61	43 79 37	45 38 42
		35 13 30	19 17 13	39 35 30	41 49 38	58 40 56	37 37 30	45 45 57	51 64 53
		438 Wa	10 ^h 20 ^m 0.764	438 Wa	10 ^h 20 ^m 0.764	451 Wa	12 ^h 00 ^m 1.404	438 Wa	10 ^h 20 ^m 0.764
		52 73 82	90 70 54	32 23 35	27 30 55	33 29 28	32 26 40	38 49 75	155 55 32
		44 31 63	102 74 57	44 31 63	102 74 57	33 48 27	127 29 21	34 38 42	81 41 60
		32 23 35	27 30 55	33 29 28	32 26 40	31 49 46	26 19 50	94 39 39	27 41 53
		55 34 55	48 32 36	25 34 34	14 20 25				

TABLE VI. (continued)

		2305	6 ^h 00 ^m	2093	6 ^h 20 ^m	2138	6 ^h 40 ^m	2335	7 ^h 00 ^m
		Wm	0.818	S1	1.127	S1	1.127	Sm	0.979
$\delta = 0^\circ$		2 0 0 0 0 0	0 0 0 0 0	1 1 0 0 0 0	0 1 0 0 0 0	0 1 0 1 1 0	1 2 2 0 1 0	0 2 2 1 0 0	2 2 0 0 0 0
		0 0 0 0 0 1	0 0 0 0 0 1	0 1 0 0 0 0	0 0 0 2 3 0	0 0 1 0 1 1	0 2 2 1 0 0	0 0 0 0 0 0	0 0 0 0 0 0
		0 1 0 0 0 0	0 0 0 0 0 0	0 0 0 2 3 0	0 0 0 1 2 1	0 0 0 0 0 0	0 1 2 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
		0 0 1 0 0 2	0 0 1 0 0 2	0 0 0 1 0 2	0 0 0 1 0 2	0 0 0 0 0 0	0 1 2 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
		0 0 2 2 0 0	0 0 2 2 0 0	0 0 1 0 4 0	0 0 1 0 4 0	0 1 2 1 0 0	0 0 0 0 0 0	0 1 2 0 0 0	0 0 0 0 0 0
		0 0 1 0 0 0	0 0 1 0 0 0	0 0 0 0 1 0	0 0 0 0 1 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0	0 0 0 0 0 0
1084	7 ^h 20 ^m	1510	7 ^h 40 ^m	1127	8 ^h 00 ^m	2149	8 ^h 20 ^m	1177	8 ^h 40 ^m
We	0.789	Sh	0.804	Se	1.048	S1	1.004	We	0.569
2 3 9 4 3 0		12 3 10 9 9 0		20 13 9 15 10 7		7 23 21 8 8 11		30 52 57 43 21 15	
4 7 3 3 2 1		13 8 4 10 7 5		4 14 12 8 8 9		8 15 18 15 15 3		42 74 51 65 40 40	
7 4 5 3 5 2		16 10 18 10 8 10		14 11 15 6 14 9		29 32 21 13 20 7		64 49 36 56 41 46	
7 4 9 8 2 0		9 11 16 16 1 6		7 19 23 13 9 6		27 22 27 20 21 6		15 20 32 31 29 37	
5 4 3 6 1 0		7 11 5 7 5 4		18 14 29 12 11 3		16 28 14 13 11 10		30 33 31 33 62 20	
6 2 2 1 2 0		15 7 13 8 5 3		8 10 16 14 19 8		18 18 19 14 16 4		25 25 24 36 33 30	
2843	9 ^h 00 ^m	1196	9 ^h 20 ^m	784	9 ^h 40 ^m	2557	10 ^h 00 ^m	440	10 ^h 20 ^m
So	1.573	We	0.734	Sb	0.901	Wo	1.086	Sa	1.216
20 23 15 21 7 7		35 58 64 48 15 25		55 54 44 66 39 36		14 27 43 37 31 38		28 35 27 45 29 8	
12 23 20 27 20 15		37 51 45 58 33 24		73 36 32 22 36 28		31 43 44 62 71 58		21 36 42 45 25 17	
17 27 9 17 24 18		57 54 43 37 23 33		66 43 40 39 22 48		65 85 54 113 83 33		30 30 38 38 62 54	
9 16 14 11 21 7		35 48 45 53 31 21		44 28 39 24 12 23		115 74 54 56 56 39		34 27 24 51 47 97	
31 27 16 17 15 11		40 51 36 47 62 41		24 35 41 24 44 23		49 60 60 38 61 23		41 34 35 47 77 59	
17 14 13 11 18 14		48 36 40 36 28 18		33 43 41 28 28 37		52 44 29 63 25 18		32 57 38 74 45 45	
446	10 ^h 40 ^m	1514	11 ^h 00 ^m	1874	11 ^h 20 ^m	1197	11 ^h 40 ^m	792	12 ^h 00 ^m
Sa	0.938	Sh	0.714	Wj	1.000	Se	1.002	Sb	0.658
45 43 52 67 53 29		85 69 45 31 35 45		61 74 67 55 67 80		31 55 47 44 23 43		71 82 137 113 84 55	
35 58 82 89 51 34		107 65 63 63 73 33		76 72 89 70 43 60		34 27 24 44 53 81		61 72 101 59 78 52	
43 83 60 49 37 39		101 87 43 38 52 49		43 62 76 72 51 61		49 42 36 32 40 33		71 84 89 102 85 74	
41 50 51 49 34 44		85 75 57 48 40 61		44 66 56 54 56 62		46 74 52 60 44 24		99 78 73 119 120 76	
30 37 49 35 26 48		50 60 58 42 42 36		60 65 47 50 51 21		44 89 133 36 44 43		54 66 77 81 88 90	
26 51 42 37 32 47		87 76 49 56 41 30		45 37 40 46 24 39		78 76 71 55 58 22		56 49 66 58 59 55	
$\delta = +5^\circ$		2137	6 ^h 00 ^m	2099	6 ^h 20 ^m	1839	6 ^h 40 ^m	2380	7 ^h 00 ^m
		S1	1.127	S1	1.127	Sj	0.916	Wm	0.876
		2 0 4 1 0 4		0 0 0 0 1 0		0 0 0 0 0 1		11 1 3 1 0 0	
		1 1 1 1 1 3		0 0 0 0 0 1		0 0 0 0 1 0		12 5 0 5 2 0	
		0 1 1 0 1 1		0 0 1 0 0 0		1 0 0 0 1 0		8 3 0 1 1 1	
		1 0 0 0 1 0 1		0 0 0 1 1 1 1		0 1 0 0 0 3		4 1 1 1 1 0	
		1 0 0 1 2 0		0 0 0 2 2 0		2 1 1 0 0 0		1 3 1 0 0 0	
		1 2 1 0 1 0		0 0 0 0 0 1		0 1 1 0 0 0		1 4 3 2 0 0	
1812	7 ^h 20 ^m	1503	7 ^h 40 ^m	2121	8 ^h 00 ^m	1485	8 ^h 20 ^m	768	8 ^h 40 ^m
Sj	1.198	Sh	0.855	W1	0.948	Wg	0.537	Sb	1.259
4 4 6 6 3 8		22 11 23 21 5 6		25 49 35 38 14 19		24 31 67 58 34 29		18 11 9 11 18 12	
7 11 4 11 7 14		21 34 24 20 5 7		38 34 41 37 27 24		20 25 54 52 25 38		17 11 16 12 8 9	
7 14 6 6 8 5		20 32 16 19 18 13		33 83 43 32 13 27		25 39 49 58 57 48		27 23 21 21 18 14	
6 5 5 6 4 1		24 26 16 16 14 8		32 28 27 22 32 20		48 44 142 91 56 43		32 30 29 18 35 25	
4 6 5 5 6 1		18 8 12 9 7 5		29 17 20 33 20 14		33 36 58 48 47 27		14 29 32 26 15 9	
1 2 7 4 2 0		21 7 10 7 6 4		24 19 8 8 14 16		25 63 62 50 33 35		12 29 33 28 15 8	
1549	9 ^h 00 ^m	1495	9 ^h 20 ^m	1199	9 ^h 40 ^m	1496	10 ^h 00 ^m	1853	10 ^h 20 ^m
Wh	0.688	Sh	1.015	We	0.857	Wj	0.821	Wj	0.763
49 54 32 32 41 25		30 43 46 55 61 33		57 53 40 26 42 33		41 61 87 46 51 58		67 40 64 69 53 47	
40 36 33 37 45 35		25 19 27 58 38 24		72 84 87 40 52 29		81 58 62 46 38 51		39 54 62 70 80 44	
32 38 44 37 34 39		22 26 28 30 26 16		58 47 53 47 37 27		42 40 37 60 48 39		62 59 71 35 51 38	
43 57 33 24 46 39		26 42 41 43 21 25		51 49 51 44 45 38		28 15 39 52 34 33		100 136 64 49 65 24	
59 49 57 61 47 18		32 32 32 16 24 30		47 31 67 61 64 48		45 33 61 80 60 36		76 110 54 67 60 41	
30 37 25 31 26 19		25 31 37 42 9 21		51 51 35 54 35 15		15 45 42 32 41 33		41 59 54 80 32 14	
1126	10 ^h 40 ^m	813	11 ^h 00 ^m	1565	11 ^h 20 ^m	1215	11 ^h 40 ^m	758	12 ^h 00 ^m
Se	0.864	Sb	1.195	Sh	0.682	We	1.106	Wb	0.906
51 53 32 43 28 54		44 36 70 56 42 22		42 52 62 39 43 55		42 71 76 48 44 25		58 37 42 40 42 76	
43 41 29 39 57 32		32 54 44 63 49 28		75 40 39 50 35 55		78 72 58 76 31 47		89 67 43 54 56 74	
55 66 35 115 61 51		43 55 52 31 40 30		72 39 30 40 51 60		98 96 99 111 58 57		80 62 44 56 89 100	
71 35 49 72 70 94		59 37 42 46 21 42		55 60 30 59 67 77		82 79 73 79 45 44		49 40 94 39 73 85	
39 59 45 53 52 55		47 63 48 34 18 36		51 69 63 133 94 56		57 47 39 51 52 29		46 36 40 61 47 60	
56 39 53 75 52 30		81 63 41 33 28 33		73 105 79 72 79 116		23 44 33 47 20 37		45 51 94 73 52 36	

TABLE VI. (continued)

$\delta = +10^\circ$										2329	6 ^h 00 ^m	2108	6 ^h 20 ^m	1826	6 ^h 40 ^m	1494	7 ^h 00 ^m
										S _m	0.979	S ₁	0.944	S _j	0.916	S _h	0.952
										0 1 1	0 1 3	1 1 0	0 1 0	1 2 1	0 0 0	7 4 7	1 8 1
										0 0 0	0 4 1	0 0 0	0 2 0	0 3 0	1 1 0	11 10 1	4 1 2
										1 0 1	1 3 0	0 0 0	1 0 1	1 3 1	0 1 2	4 10 7	8 5 1
										0 0 4	4 4 1	0 0 0	0 0 0	2 2 1	0 0 0	7 11 3	0 1 3
										0 0 2	1 1 0	0 0 0	0 1 0	0 0 1	0 0 0	3 7 3	5 8 3
										2 0 1	0 1 1	0 0 0	0 1 0	0 0 1	0 0 2	13 4 2	2 1 0
										708	7 ^h 20 ^m	718	7 ^h 40 ^m	2132	8 ^h 00 ^m	2152	8 ^h 20 ^m
										W _b	0.697	S _b	1.003	S ₁	0.852	S ₁	1.447
16	18	16	9	7	14	23	21	14	20	24	7	6	13	27	24	21	20
16	15	15	6	6	11	14	17	15	12	16	7	21	32	23	25	34	17
17	14	13	7	18	2	26	24	21	17	27	9	27	16	43	35	21	25
16	9	20	20	18	8	58	26	62	44	16	16	19	35	28	52	20	48
6	7	8	2	3	1	29	16	26	23	10	6	15	37	44	42	30	20
8	8	9	11	9	12	17	11	33	12	10	5	26	46	35	35	10	14
2139	9 ^h 00 ^m	2134	9 ^h 20 ^m	1131	9 ^h 40 ^m	1567	10 ^h 00 ^m	1866	10 ^h 20 ^m								
S ₁	1.041	S ₁	1.213	S _e	0.724	S _h	1.256	S _j	0.861								
25	26	26	26	28	19	22	50	29	28	27	17	34	28	29	27	23	39
22	31	29	44	49	23	23	17	35	27	24	22	64	63	66	69	75	46
32	30	42	25	33	23	66	43	17	27	33	17	71	52	44	51	66	81
24	23	32	27	43	35	66	26	22	25	30	19	35	68	47	36	52	67
25	25	33	30	34	25	37	27	53	29	30	19	56	51	57	28	51	46
36	30	25	19	30	17	22	23	32	39	51	24	61	54	47	24	39	27
1486	10 ^h 40 ^m	753	11 ^h 00 ^m	1163	11 ^h 20 ^m	1573	11 ^h 40 ^m	1149	12 ^h 00 ^m								
W _g	0.788	S _b	0.892	S _e	0.987	S _h	0.819	S _e	0.999								
79	77	72	87	80	75	36	27	53	70	71	68	48	30	41	53	43	28
96	82	80	81	79	103	55	43	36	39	43	50	23	32	28	26	37	42
108	60	67	67	83	40	45	51	39	107	50	71	53	57	48	44	35	62
132	101	76	64	41	30	69	61	61	98	102	99	28	50	47	43	36	60
94	47	71	57	38	65	63	87	55	99	73	78	50	38	57	48	42	50
53	68	43	66	39	63	56	49	98	76	65	40	44	39	48	31	38	43
$\delta = +15^\circ$										1819	6 ^h 00 ^m	2142	6 ^h 20 ^m	1466	6 ^h 40 ^m	1830	7 ^h 00 ^m
										S _j	0.916	S ₁	1.127	S _g	0.940	W _j	1.009
										2 0 0	0 0 1	0 0 0	0 0 0	4 3 7	0 0 0	5 4 6	4 2 2
										0 1 0	0 2 4	0 0 0	0 0 1	4 3 5	0 0 0	1 5 9	6 10 6
										0 0 0	2 3 3	0 0 0	0 0 0	6 1 3	1 1 0	6 12 15	6 11 6
										1 0 0	0 1 2	0 0 0	0 1 0	5 2 4	4 1 0	4 5 11	12 10 8
										0 0 0	0 2 1	0 0 0	0 0 0	0 0 1	1 0 0	8 16 13	3 7 0
										0 0 0	0 0 2	0 0 0	0 0 0	1 2 1	1 0 0	11 4 5	1 4 2
										783	7 ^h 20 ^m	1106	7 ^h 40 ^m	2341	8 ^h 00 ^m	1551	8 ^h 20 ^m
										S _b	0.849	S _e	0.769	S _m	1.606	W _h	0.613
17	23	29	20	21	4	31	49	42	33	38	19	29	26	17	10	14	11
15	7	9	3	8	2	44	31	17	33	24	13	22	28	27	32	14	29
22	16	11	10	10	9	41	25	18	10	13	15	18	40	37	34	15	17
12	22	11	12	10	8	59	22	20	26	13	11	10	20	15	11	31	22
18	20	5	8	10	4	35	16	19	18	21	24	11	13	8	12	15	14
13	10	13	6	4	9	27	21	17	16	24	4	6	7	13	8	7	12
1468	9 ^h 00 ^m	1831	9 ^h 20 ^m	791	9 ^h 40 ^m	1843	10 ^h 00 ^m	437	10 ^h 20 ^m								
W _g	0.829	S _j	1.069	S _b	0.892	S _j	0.849	S _a	1.129								
54	53	96	77	68	36	24	25	34	30	62	35	34	40	26	39	48	18
78	107	49	70	61	28	27	24	35	40	50	52	37	32	40	37	28	28
75	44	64	55	70	59	28	37	39	56	74	24	46	37	50	44	37	27
52	97	85	62	66	43	28	44	48	63	33	23	22	56	49	39	45	40
59	58	70	51	41	30	28	46	72	31	30	32	49	43	27	49	37	24
31	35	27	25	32	15	24	50	28	31	16	19	59	39	21	29	30	25
1552	10 ^h 40 ^m	772	11 ^h 00 ^m	795	11 ^h 20 ^m	1875	11 ^h 40 ^m	1132	12 ^h 00 ^m								
S _h	0.808	W _b	0.765	S _b	0.856	S _j	0.824	S _e	0.810								
92	71	41	59	34	33	46	54	57	83	92	73	86	95	68	57	67	35
74	83	84	73	51	68	58	110	77	73	83	95	83	75	52	48	54	70
52	59	48	40	87	40	43	82	65	75	50	48	79	71	55	48	42	46
74	54	80	49	38	42	107	106	56	58	77	65	104	54	46	50	56	78
67	66	52	45	53	70	86	89	78	59	54	56	37	33	31	46	38	66
84	74	79	68	68	77	46	49	72	96	77	54	46	31	35	66	44	25

TABLE VI. (continued)

		1462	6 ^h 00 ^m	2353	6 ^h 20 ^m	1475	6 ^h 40 ^m	2331	7 ^h 00 ^m
		Sg	0.940	Sm	0.991	Sg	1.212	Sg	0.861
$\delta = +20^\circ$		2 0 0	1 3 0	5 2 1	0 1 0	6 4 4	3 9 5	6 9 18	16 11 4
		1 0 0	0 0 2	5 3 0	1 0 0	6 2 1	2 2 3	14 30 33	14 8 7
		0 0 0	0 1 0	4 1 0	0 0 0	5 14 1	5 0 2	10 23 24	12 9 13
		1 0 0	0 1 0	2 0 0	0 0 0	10 6 3	1 1 1	23 17 30	13 16 15
		0 1 1	1 0 4	2 4 0	0 0 0	2 5 1	2 0 1	19 11 28	11 11 3
		1 2 0	0 1 3	0 1 0	0 0 0	1 3 5	2 0 1	6 4 8	4 2 4
		735	7 ^h 20 ^m	2491	8 ^h 00 ^m	1463	8 ^h 20 ^m	777	8 ^h 40 ^m
		Wb	0.830	Sn	1.091	Sg	0.996	Sb	1.096
16 20 30	32 21 12	16 22 30	17 16 15	19 19 25	13 20 14	47 35 33	51 40 28	22 30 41	27 17 22
16 22 16	31 24 14	23 15 18	13 20 22	25 27 17	19 16 25	41 40 52	96 41 42	27 35 32	18 21 16
30 31 38	35 17 21	15 22 11	18 12 14	24 31 37	28 20 26	25 28 64	54 50 34	41 27 27	17 19 30
36 28 33	36 19 37	22 39 19	22 51 20	24 26 30	22 35 16	50 35 72	29 24 27	42 71 35	33 38 41
70 50 32	35 30 16	25 60 49	43 33 49	30 27 20	29 28 18	36 51 66	47 17 35	75 58 30	53 42 35
22 18 34	20 16 4	15 29 26	22 25 8	28 20 25	18 14 14	26 79 44	21 10 29	51 39 30	39 39 28
		1476	9 ^h 00 ^m	2573	9 ^h 40 ^m	1135	10 ^h 00 ^m	1817	10 ^h 20 ^m
		Sg	0.746	So	1.252	We	0.850	Sj	0.966
44 48 51	51 56 32	17 22 75	19 20 17	25 33 47	41 33 25	41 26 37	30 28 24	29 54 54	35 38 26
33 63 49	79 37 48	27 34 32	19 15 12	51 48 46	29 29 30	57 58 41	36 41 58	42 63 84	67 46 26
43 58 74	94 53 39	15 22 21	38 54 18	51 70 15	32 18 21	95 42 27	44 86 69	44 61 77	75 43 60
46 62 109	55 39 36	16 15 25	23 44 29	51 27 23	29 21 17	108 115 72	94 88 87	53 45 54	59 47 75
46 70 106	57 61 69	21 26 33	32 27 12	35 30 19	12 13 17	62 109 84	67 47 40	41 65 60	46 22 43
60 66 114	78 64 47	12 19 22	30 42 19	13 22 23	32 30 10	61 54 44	74 45 23	33 55 76	46 35 39
		2150	10 ^h 40 ^m	2140	11 ^h 20 ^m	809	11 ^h 40 ^m	751	12 ^h 00 ^m
		Sl	1.146	Sl	1.378	Sb	0.969	Sb	1.002
60 86 49	33 37 30	28 55 38	37 43 49	55 36 38	36 22 32	77 94 65	80 80 60	67 70 97	72 76 77
47 35 42	39 47 39	23 59 38	30 54 57	83 74 30	44 29 24	74 99 129	63 132 64	66 48 85	98 89 119
40 48 48	20 45 35	27 38 43	34 63 37	59 50 49	40 30 22	100 82 213	92 90 85	63 42 142	76 36 106
55 41 37	44 29 27	49 58 66	44 56 43	33 54 65	66 40 44	44 65 141	71 40 32	74 51 59	45 50 50
54 60 59	51 32 31	13 32 55	46 35 47	57 41 28	31 34 10	61 41 44	83 99 86	55 49 50	54 55 47
58 66 25	33 21 19	35 25 38	51 60 44	36 38 27	37 39 13	34 35 52	71 58 45	48 42 43	39 41 39

(2) If the absorbing material lies in a very thin layer of total optical depth $2ak$ distributed symmetrically on either side of the fundamental plane, an external object viewed from this plane and normal to it will be reduced in apparent brightness by a factor e^{-ak} , or by $\Delta m_1 = -2.5 \log e^{-ak}$. On the other hand, if the nebula is viewed normal to its fundamental plane from an external point, the absorbing layer will cause a reduction in brightness by a factor $(1 + e^{-2ak})/2$, or

$$\Delta m_2 = -2.5 \log \frac{1 + e^{-2ak}}{2}. \quad (3)$$

Therefore,

$$\Delta m_1 - \Delta m_2 = 2.5 \log \cosh ak. \quad (4)$$

But,

$$ak = 0.921 \Delta m_1.$$

Therefore,

$$\Delta m_1 - \Delta m_2 = 2.5 \log \cosh (0.921 \Delta m_1), \quad (5)$$

and when $\Delta m_1 = 0.46$, $\Delta m_2 = 0.37$.

(3) It is assumed that:

a) throughout the nebula the ratio, γ , of volume emission to absorption coefficients is constant,

b) these coefficients are distributed uniformly on either side of the fundamental plane.

Let τ_0 be the optical depth in absorption of an external object seen from and normal to the fundamental plane. Then

$$\Delta m_1 = 2.5 \log e^{\tau_0} = 1.086 \tau_0. \quad (6)$$

Let τ be the optical depth in absorption of a point in the nebula as viewed from an external position normal to the fundamental plane. If the point is in that plane, $\tau = \tau_0$. Because of (b) the total optical thickness normal to the fundamental plane is $2\tau_0$. From (a) it is clear that the reduction in brightness due to absorption as seen from the external position is in the ratio

$$\frac{\gamma \int_0^{2\tau_0} e^{-\tau} d\tau}{\gamma \int_0^{\tau_0} d\tau} = \frac{1 - e^{-2\tau_0}}{2\tau_0} \quad (7)$$

but, since $\Delta m_1 = 1.086 \tau_0$, we find

$$\Delta m_2 = 2.5 \log \frac{1.842 \Delta m_1}{1 - e^{-1.842 \Delta m_1}} \quad (8)$$

and when $\Delta m_1 = 0.46$, $\Delta m_2 = 0.42$.

TABLE VII. (continued)

		785	12 ^h 00 ^m	1598	12 ^h 20 ^m	1213	12 ^h 40 ^m	1613	13 ^h 00 ^m
		Sb	0.974	Wh	0.681	Se	1.300	Wh	0.645
		50 88 79 65 67 48		43 49 109 55 76 83		26 22 20 34 22 20		69 87 73 69 52 47	
		47 68 62 62 50 36		55 102 158 94 68 104		37 25 34 19 25 39		91 72 66 119 82 73	
$\delta = +30^\circ$		109 62 62 91 61 26		41 102 72 120 127 85		43 49 40 22 24 35		111 74 59 75 81 83	
		82 78 66 80 40 55		98 91 67 94 86 116		37 40 25 28 37 64		76 74 114 119 94 99	
		73 72 124 55 36 57		69 104 101 165 116 71		63 39 46 44 43 30		93 102 138 395 156 62	
		49 55 37 37 74 43		90 105 112 128 86 68		75 72 34 46 72 34		76 69 90 215 190 128	
1881		1581	13 ^h 40 ^m	2203	14 ^h 00 ^m	1886	14 ^h 20 ^m	1599	14 ^h 40 ^m
Wj		Wh	0.757	Sm	0.666	Sj	1.151	Sh	0.703
80 113 116	76 64 83	64 110 55 60 86 96		45 74 49 79 51 89		35 45 43 39 29 37		55 67 59 35 30 41	
85 95 108 128	79 53	83 65 72 98 68 88		71 40 39 38 81 53		50 72 46 38 53 32		100 84 74 55 58 65	
62 71 60 81 85 60		74 77 155 116 62 60		68 73 59 90 69 62		65 68 58 79 51 46		87 118 65 86 79 77	
81 44 44 59 86 71		128 158 110 120 118 54		78 77 89 81 59 138		63 53 79 69 64 48		74 83 50 60 83 44	
60 58 42 57 82 83		125 188 91 98 85 50		125 89 113 264 135 123		64 49 53 57 60 63		75 82 48 55 78 62	
40 47 83 47 37 43		116 162 87 86 57 47		101 67 85 84 93 96		30 38 45 50 45 41		49 70 67 76 57 41	
1219		1870	15 ^h 20 ^m	1882	15 ^h 40 ^m	1214	16 ^h 00 ^m	1228	16 ^h 20 ^m
Sf		Sj	0.858	Sj	0.915	Se	1.089	Wf	0.710
42 46 51 79 51 46		42 70 107 109 66 49		43 62 42 58 42 61		47 37 38 40 35 30		40 53 43 34 46 76	
89 54 80 51 82 75		144 69 177 102 87 74		40 45 25 49 119 64		82 51 58 49 37 23		67 42 35 58 97 73	
52 63 64 50 61 93		106 140 195 181 87 95		59 56 50 67 74 71		89 65 64 37 54 45		42 56 91 93 111 95	
39 72 69 73 41 56		65 201 187 122 59 51		75 83 36 42 63 138		96 54 55 58 63 60		53 52 48 100 86 132	
89 62 123 75 69 72		98 136 182 162 110 63		71 108 73 68 63 72		53 58 56 58 76 72		58 74 69 69 86 94	
46 86 60 45 37 27		66 74 169 140 77 56		148 35 53 41 61 40		66 69 60 117 97 69		68 53 58 51 78 57	
1659		843	17 ^h 00 ^m	1897	17 ^h 20 ^m	1682	17 ^h 40 ^m	1923	18 ^h 00 ^m
Si		Wb	0.615	Sj	1.144	Wi	0.696	Sk	1.449
28 44 33 51 20 17		58 51 54 111 75 64		13 26 28 24 20 24		30 30 33 31 21 25		6 8 13 3 28 28	
36 35 46 32 47 26		52 81 94 63 65 65		29 47 41 24 35 34		32 33 28 22 27 40		13 12 15 33 20 19	
57 51 42 40 22 16		52 65 68 48 79 66		26 22 53 45 35 32		37 35 30 16 27 22		18 30 20 12 29 14	
52 56 44 52 32 28		57 55 59 56 68 72		11 25 27 34 34 24		32 37 24 32 34 32		15 15 44 25 23 26	
31 37 32 66 40 19		43 58 91 84 50 56		10 18 17 42 28 46		24 31 27 15 10 8		15 21 27 31 23 13	
38 77 63 39 36 33		40 54 30 82 60 54		14 25 49 34 21 29		26 35 22 14 8 15		8 10 13 21 20 3	
		1200	12 ^h 00 ^m	1847	12 ^h 20 ^m	1221	12 ^h 40 ^m	1205	13 ^h 00 ^m
		Se	0.855	Wj	0.815	We	0.850	We	0.604
		51 92 68 66 45 39		40 45 38 41 37 83		22 21 24 28 23 27		67 61 56 79 56 40	
		87 129 66 88 86 71		53 72 52 50 66 92		41 34 47 34 32 48		70 69 93 76 78 55	
$\delta = +35^\circ$		61 68 58 77 60 52		55 65 34 37 54 57		50 45 31 39 51 33		61 83 68 65 88 75	
		61 50 46 60 59 42		80 56 38 52 54 51		35 34 51 50 64 31		109 100 75 40 35 45	
		62 90 85 101 70 46		62 53 63 71 45 78		57 53 57 68 47 43		63 72 85 57 54 49	
		48 85 88 57 54 39		33 43 107 55 50 69		34 34 34 46 37 38		58 71 53 63 41 38	
1624		1858	13 ^h 40 ^m	2186	14 ^h 00 ^m	1859	14 ^h 20 ^m	2418	14 ^h 40 ^m
Wh		Sj	0.791	Wm	0.771	Sj	0.809	Wm	1.069
81 88 63 51 44 30		99 55 50 41 99 131		59 85 70 73 110 56		32 76 64 58 44 48		31 26 34 36 25 37	
121 63 67 51 58 52		126 88 69 55 141 62		110 63 63 74 82 68		76 81 71 72 89 64		23 40 44 41 49 53	
101 60 51 51 58 71		125 89 56 60 104 43		87 65 63 96 124 92		80 82 77 76 45 38		32 53 49 36 56 59	
127 84 47 103 91 96		126 86 49 62 116 65		56 44 32 81 106 69		64 65 61 56 37 37		37 68 41 31 40 40	
122 67 97 129 76 55		149 97 49 53 101 76		39 66 92 90 139 94		47 66 60 33 26 27		52 63 50 35 29 56	
92 122 112 88 58 72		62 104 45 49 69 84		48 77 73 82 61 81		30 36 53 39 21 40		43 44 43 19 19 23	
495		1206	15 ^h 00 ^m	1625	15 ^h 40 ^m	1582	16 ^h 00 ^m	1268	16 ^h 20 ^m
Sa		Se	0.962	Sh	0.933	Wh	0.809	Sf	0.979
33 55 40 36 32 32		38 29 59 37 28 51		28 46 32 30 53 17		68 87 70 62 42 51		39 28 75 72 58 71	
47 105 111 54 36 33		46 38 65 50 44 80		58 50 114 33 44 32		37 66 56 52 59 36		43 43 52 48 38 62	
84 70 90 48 48 51		60 54 37 39 67 50		65 54 81 58 48 38		70 55 38 88 75 57		37 44 64 101 73 41	
51 71 41 57 50 41		67 52 36 60 70 82		82 66 93 66 47 37		38 60 64 74 84 60		26 34 47 85 47 55	
36 71 29 45 46 44		82 80 92 105 67 74		63 90 67 69 56 51		59 79 121 79 88 72		34 32 45 46 59 45	
49 43 48 81 43 28		50 66 85 81 46 36		48 57 58 68 53 57		54 60 46 52 54 48		19 33 36 27 37 39	
1594		883	17 ^h 00 ^m	1899	17 ^h 20 ^m	1900	17 ^h 40 ^m	1922	18 ^h 00 ^m
Wh		Sb	0.775	Wj	0.764	Sj	1.258	Wk	0.661
69 42 66 86 60 33		53 47 63 51 44 27		24 25 14 24 35 40		31 20 16 23 22 16		38 35 22 29 72 39	
67 54 52 77 70 64		46 50 79 39 48 36		67 50 34 36 46 40		9 9 21 22 52 21		39 37 18 38 25 17	
64 68 65 53 58 56		71 64 92 53 43 47		43 35 38 46 57 51		9 12 23 54 18 18		42 23 29 26 18 17	
73 64 58 74 41 45		64 148 121 68 44 31		39 39 48 68 52 89		15 26 22 35 19 10		37 15 18 18 36 29	
71 61 53 72 63 50		57 93 180 83 54 39		50 23 48 41 59 40		14 27 40 20 24 8		24 23 22 24 26 23	
71 85 64 96 47 32		29 42 41 87 50 44		23 32 36 40 23 29		34 24 26 17 18 7		19 21 27 14 48 27	

TABLE VII. (continued)

	1525 12 ^h 00 ^m		1600 12 ^h 24 ^m		1570 12 ^h 48 ^m		461 13 ^h 12 ^m	
	Sh	0.715	Sh	0.704	Wh	1.190	Sa	0.932
$\delta = +40^\circ$	100 72 99 67 56 63		79 53 43 43 80 41		20 34 21 37 61 39		44 57 69 47 29 21	
	129 113 71 84 53 44		69 66 72 65 83 59		29 25 38 56 42 31		41 56 85 53 36 41	
	66 90 69 35 41 44		37 57 66 56 59 56		102 81 64 36 31 30		44 59 53 76 130 97	
	58 71 108 62 45 28		50 40 48 41 59 36		49 56 59 36 31 30		56 64 67 169 58 92	
	41 58 56 65 29 41		63 45 45 35 45 55		50 40 41 23 25 22		65 54 80 51 49 66	
	36 72 57 50 37 38		40 51 47 36 28 36		29 37 20 18 34 18		75 73 43 41 17 37	
	2187 14 ^h 00 ^m		2196 14 ^h 24 ^m		1638 14 ^h 48 ^m		1611 15 ^h 12 ^m	
	Sm	0.838	Sm	1.055	Wh	0.772	Sh	0.816
1896 13 ^h 36 ^m								
Sj	1.003							
8 28 25 49 36 55	50 59 30 33 23 43		28 27 34 33 42 33		51 30 41 29 29 23		43 66 45 52 59 43	
10 56 36 53 37 29	80 67 40 35 39 45		27 27 38 30 41 42		24 42 40 42 38 18		46 58 62 61 45 42	
15 52 42 32 53 51	66 66 64 33 80 63		26 32 38 67 43 32		59 40 41 26 39 42		35 40 46 52 46 42	
6 48 57 89 58 58	65 68 67 89 80 77		30 20 18 47 38 45		47 57 49 34 36 51		49 43 33 36 40 43	
15 57 56 98 86 31	48 59 51 67 96 93		32 20 40 42 33 37		77 77 51 53 34 42		41 56 40 32 54 47	
10 37 25 59 96 49	35 41 49 36 97 29		47 25 52 35 23 27		54 53 52 45 23 28		36 53 35 61 61 65	
	1593 16 ^h 00 ^m		1575 16 ^h 24 ^m		1904 16 ^h 48 ^m		1674 17 ^h 12 ^m	
	Sh	0.828	Wh	0.845	Sj	1.463	Si	1.019
8 53 46 44 44 33	39 71 46 37 37 42		43 54 51 52 56 39		39 28 33 27 30 27		36 31 58 62 58 32	
13 57 79 68 47 52	48 53 58 44 78 27		37 66 135 83 72 38		49 27 36 24 14 27		21 47 67 46 60 39	
10 50 56 62 45 44	65 33 54 59 59 40		92 95 169 86 56 46		45 46 43 48 36 40		25 41 52 83 57 42	
6 54 48 49 37 54	73 62 49 45 64 27		75 69 207 89 41 45		56 65 45 43 37 17		32 52 81 109 62 48	
6 34 63 64 46 50	84 75 46 37 41 43		34 30 56 88 64 46		44 37 35 24 16 15		15 35 83 76 43 31	
0 40 41 77 28 35	73 93 65 49 32 42		45 48 37 76 49 59		31 23 23 10 33 20		13 16 32 38 33 32	
	1652 18 ^h 00 ^m							
	Si	0.923						
1898 17 ^h 36 ^m								
Sj	0.863							
3 24 19 26 21 25	34 40 59 61 32 16							
6 22 31 42 31 35	46 43 32 45 44 26							
4 34 49 39 37 26	17 23 25 35 50 48							
7 74 47 27 38 34	27 31 40 49 33 42							
9 58 38 30 30 31	30 26 36 34 38 45							
8 26 40 32 30 11	33 17 18 30 60 22							
	1892 12 ^h 00 ^m		1596 12 ^h 24 ^m		1860 12 ^h 48 ^m		1619 13 ^h 12 ^m	
	Sj	1.234	Wh	1.171	Sj	0.635	Wh	0.843
$\delta = +45^\circ$	40 28 36 37 32 28		28 40 40 38 48 37		57 52 54 50 49 37		44 27 79 68 62 39	
	39 27 32 50 27 20		28 29 27 67 34 20		55 70 57 50 43 36		36 34 39 86 40 37	
	49 39 24 44 26 26		36 21 40 46 37 21		94 56 102 48 69 45		51 58 73 111 55 34	
	71 49 24 42 67 42		28 31 18 38 56 37		89 53 50 46 59 41		59 55 91 59 55 25	
	37 30 79 51 54 24		36 50 45 42 39 39		82 57 43 39 49 59		48 68 118 93 94 39	
	22 35 47 22 14 19		39 37 23 29 45 22		54 56 38 58 84 72		63 88 97 58 48 26	
	2190 14 ^h 00 ^m		1893 14 ^h 24 ^m		1905 14 ^h 48 ^m		1862 15 ^h 12 ^m	
	Sm	1.030	Sj	1.285	Sj	0.953	Wj	0.662
9 36 28 34 24 15	22 32 25 42 33 25		11 12 24 44 18 27		35 59 43 37 34 24		43 31 62 48 78 108	
5 41 42 28 34 21	29 33 40 53 83 50		27 36 37 40 30 28		32 27 36 48 62 58		46 43 45 62 47 35	
15 59 36 36 29 27	33 33 26 40 38 41		17 41 31 27 26 33		49 44 22 21 35 79		82 75 76 38 49 44	
17 38 45 47 35 40	28 36 40 25 40 32		32 25 30 57 27 39		25 34 17 28 28 21		73 92 88 38 42 52	
12 27 34 41 15 28	43 35 33 40 45 37		23 36 35 27 28 45		40 26 14 27 24 36		67 60 64 47 65 53	
9 25 15 35 22 32	37 31 17 26 22 15		18 21 29 22 32 33		50 26 28 30 36 23		51 51 50 52 72 50	
	1260 16 ^h 00 ^m		1655 16 ^h 24 ^m		1670 16 ^h 48 ^m		1661 17 ^h 12 ^m	
	Wf	0.799	Si	0.992	Wi	0.817	Si	0.910
1 28 41 27 24 18	27 38 52 43 28 36		40 38 32 34 33 26		35 44 40 29 40 37		33 22 32 32 37 39	
12 31 49 31 37 27	38 25 29 44 46 32		45 34 40 40 36 34		38 55 57 47 54 45		40 41 26 26 30 40	
15 47 35 40 50 35	42 37 49 53 54 46		40 37 30 43 24 25		32 44 63 46 29 31		40 42 53 31 35 46	
15 43 37 26 52 32	34 60 37 35 25 33		54 63 53 51 33 41		39 53 53 66 44 89		62 33 72 52 54 43	
15 36 22 44 32 19	55 48 45 32 32 41		38 38 43 45 53 36		39 60 58 44 45 26		44 39 101 65 48 31	
12 35 24 23 26 15	36 53 42 40 48 43		37 47 56 37 39 27		72 76 66 51 59 41		48 33 67 72 60 24	
	501 18 ^h 00 ^m							
	Sa	1.054						
1908 17 ^h 36 ^m								
Sj	0.886							
8 22 41 52 28 17	35 33 44 30 18 21							
7 27 31 27 40 46	26 58 36 34 24 9							
14 49 20 32 20 29	23 26 62 49 47 36							
18 19 22 26 36 20	14 24 55 43 32 23							
13 17 35 44 42 31	29 26 20 30 35 16							
17 23 27 27 29 24	24 29 50 44 20 16							

TABLE VII. (continued)

		2159	12 ^h 00 ^m	1850	12 ^h 24 ^m	1861	12 ^h 48 ^m	1869	13 ^h 12 ^m
		Wl	0.861	Sj	0.771	Wj	0.660	Sj	0.890
$\delta = +50^\circ$		48 34 86	66 60 45	43 51 65	57 42 42	92 72 78	76 55 47	25 53 68	44 48 46
		55 84 120	81 59 46	84 67 75	57 76 107	100 118 103	100 65 89	23 43 58	79 68 78
		40 72 75	73 65 39	69 126 65	66 51 64	110 130 76	98 119 132	33 42 27	72 71 57
		68 52 49	45 70 23	51 63 67	80 101 53	111 72 83	75 61 66	37 38 49	92 89
		53 53 66	49 56 25	56 87 44	54 72 40	87 97 78	68 58 83	34 48 63	87 51
		47 53 49	46 44 25	50 59 54	53 59 39	84 84 75	52 71 52	53 50 101	79 66 46
		1894	13 ^h 36 ^m	2192	14 ^h 00 ^m	2198	14 ^h 24 ^m	1559	14 ^h 48 ^m
		Wj	0.721	Sm	0.977	Sm	0.771	Wh	0.659
		46 36 51	42 35 45	41 35 34	37 32 29	34 59 33	34 55 46	49 60 46	46 47 52
		29 32 36	38 30 46	34 18 35	34 36 31	50 66 39	47 31 23	32 37 70	46 82 46
		54 65 54	55 32 39	47 37 36	37 38 36	52 75 46	40 49 55	44 81 75	55 55 69
		36 48 67	33 24 30	30 44 48	66 33 29	38 54 61	66 48 55	78 120 104	54 60 78
		43 72 65	41 36 36	28 41 54	40 20 39	53 66 41	152 32 51	75 97 58	46 66 47
		35 45 51	64 52 37	27 23 21	32 22 25	45 44 50	60 34 33	76 141 86	80 56 49
		479	15 ^h 36 ^m	2215	16 ^h 00 ^m	1252	16 ^h 24 ^m	1669	16 ^h 48 ^m
		Wa	0.835	Sm	1.299	Wf	0.924	Si	0.778
		32 44 38	36 18 17	35 53 43	35 20 10	25 39 26	59 57 50	44 31 44	56 38 53
		35 37 39	33 41 37	40 48 26	28 24 13	60 43 55	77 49 32	48 41 45	83 70 42
		66 35 37	33 36 38	86 36 30	29 37 19	38 101 77	74 101 36	57 45 43	46 42 101
		30 32 31	37 29 42	45 48 39	26 15 15	56 58 55	47 91 72	43 50 44	50 57 56
		43 37 68	33 39 60	18 45 43	38 32 13	45 44 94	40 42 45	38 50 33	58 65 31
		33 47 59	39 18 17	10 22 27	29 24 17	53 37 44	46 29 30	43 50 53	47 53 41
		1684	17 ^h 36 ^m	1283	18 ^h 00 ^m				
		Si	1.182	Wf	0.857				
		39 23 20	19 22 30	16 23 37	24 34 22				
		47 32 46	30 27 31	20 24 26	37 61 36				
		13 26 38	44 32 33	23 31 28	36 23 20				
		17 24 22	34 23 27	36 68 15	24 24 18				
		38 24 24	45 17 20	24 30 23	46 34 20				
		18 18 33	35 18 5	44 38 36	42 25 19				
		2171	11 ^h 44 ^m	1589	12 ^h 16 ^m	1867	12 ^h 48 ^m	478	13 ^h 20 ^m
		Si	1.395	Wh	0.706	Sj	1.017	Wa	0.756
$\delta = +55^\circ$		35 31 31	43 50 50	71 65 80	74 96 72	35 36 42	61 45 41	65 59 77	49 44 32
		59 70 71	50 60 57	58 64 86	71 104 166	73 36 12	33 26 43	92 51 39	62 80 51
		50 66 85	70 107 45	67 37 83	65 70 113	74 72 62	73 59 22	103 79 77	55 58 87
		57 40 67	40 38 49	35 75 53	70 93 75	77 51 32	36 28 28	74 95 63	77 76 85
		25 61 62	43 24 19	78 86 67	64 71 64	66 53 71	49 60 47	67 53 55	87 95 69
		65 28 45	31 23 19	42 64 52	37 58 84	57 51 40	38 24 32	46 34 67	73 46 41
		2179	13 ^h 52 ^m	1907	14 ^h 24 ^m	1222	14 ^h 56 ^m	1620	15 ^h 28 ^m
		Si	1.751	Wj	0.748	Si	1.028	Wh	0.741
		23 24 30	28 18 22	31 39 63	38 48 45	36 22 20	34 48 44	31 63 55	49 66 22
		5 33 21	39 41 22	54 68 46	62 27 58	43 32 46	27 57 46	71 88 62	59 59 42
		61 36 31	13 56 36	78 86 69	39 77 98	30 35 52	62 74 62	81 51 51	57 45 45
		37 19 24	26 20 15	56 46 93	57 67 83	32 46 68	85 42 36	70 68 63	65 36 55
		28 15 15	20 13 12	31 82 53	41 63 84	23 36 62	53 14 29	71 63 61	66 27 31
		22 19 13	11 22 15	45 69 38	38 60 53	37 42 32	27 31 23	52 48 46	37 16 28
		1657	16 ^h 32 ^m	1671	17 ^h 04 ^m	1643	17 ^h 36 ^m	886	18 ^h 08 ^m
		Wi	0.783	Si	0.894	Wh	0.792	Sb	1.097
		25 32 40	57 102 54	102 185 78	40 35 29	55 52 39	24 57 97	30 15 32	76 50 19
		35 49 49	53 79 116	47 37 81	52 26 30	51 63 49	50 43 36	19 37 69	84 43 44
		43 38 90	96 86 75	45 26 44	47 25 25	65 92 67	48 26 40	32 33 19	29 36 29
		82 49 62	44 87 70	33 35 31	29 56 59	76 70 40	34 47 23	19 17 31	25 43 37
		81 35 58	36 72 58	57 37 49	31 46 62	56 69 48	64 64 35	17 23 39	35 22 37
		57 46 50	43 52 73	40 49 28	21 15 42	44 39 32	39 44 26	17 25 16	24 33 25

In our galaxy the obscuring matter no doubt has a distribution somewhere intermediate between that represented by cases (2) and (3). Holmberg's determination of $\Delta m_2 = 0.43$ for Sa. and Sb nebulae is reasonably consistent with our values calculated from $\Delta m_1 = 0.46$.

From this discussion of galactic extinction in the photographic region it appears that the commonly used value 0.25 mag. is very probably too low, and that it may well be greater than 0.25 mag. On the other hand, our value of 0.46 mag. could be too high, because of the increased likelihood

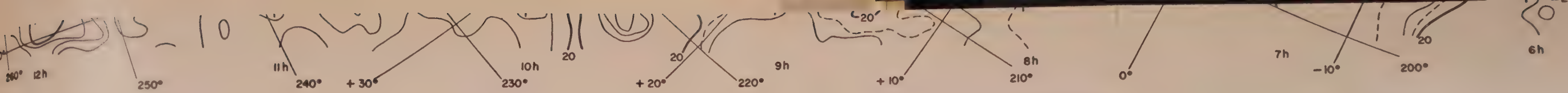


Figure 3. Equal surface density contours for nebulae in Area II, based on smoothed counts by 1° squares.

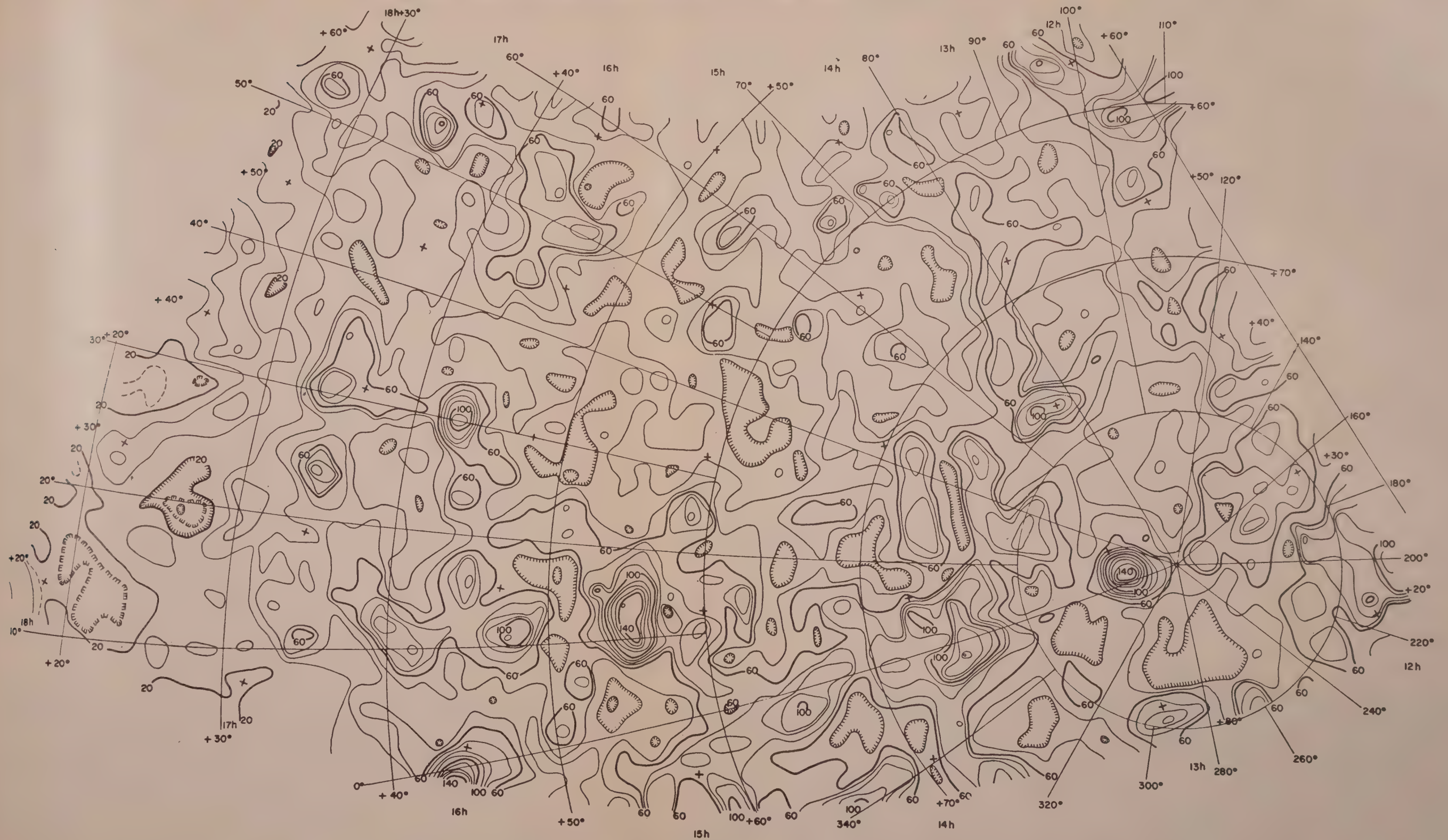


Figure 4. Equal surface density contours for nebulae in Area VII, based on smoothed counts by 1° squares.

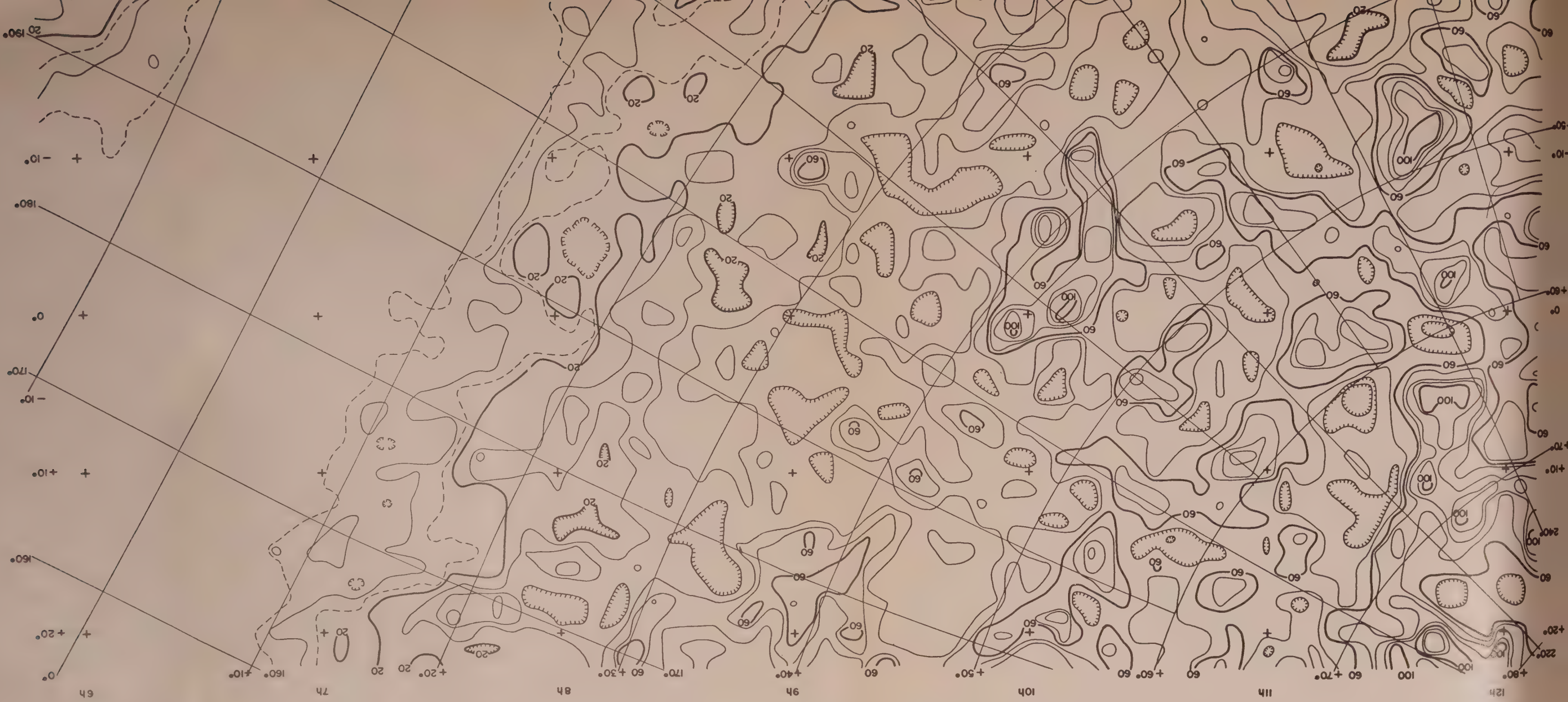


TABLE VII. (continued)

$\delta = +60^\circ$	1885 12 ^h 00 ^m		1520 12 ^h 36 ^m		1622 13 ^h 12 ^m		1901 13 ^h 48 ^m	
	Sj	0.894	Sh	0.932	Sh	0.781	Sj	0.990
	82 59 79	49 52 55	65 51 35	26 65 49	55 51 74	41 56 52	30 41 28	31 40 24
	97 78 56	35 36 56	42 30 31	33 70 76	57 62 61	64 71 23	27 32 28	45 39 44
	56144	76 49 39 66	33 49 35	33 44123	103 49 58	60 37 41	38 35 19	43 44 59
	36 97	87 62 55 50	75 53 34	51 32 55	140 73 57	35 53 37	37 59 41	44 67 48
	50 73	80 83 51 61	41 32 46	48 65 50	62 86 84	40 50 21	53 52 52	47 47 46
	51 66	70 51 63 51	34 57 40	56 45 41	71 135 72	41 52 26	44 31 31	37 40 40
	1919 14 ^h 24 ^m		1226 15 ^h 00 ^m		1651 15 ^h 36 ^m		1264 16 ^h 12 ^m	
	Sj	0.973	Si	0.897	Si	1.016	Wi	0.991
	34 41 22	38 25 40	36 33 53	28 44 45	49 44 34	39 33 20	37 25 40	31 50 34
	39 56 37	27 31 31	37 56 54	32 32 46	70 53 61	47 30 25	26 11 60	49 52 53
	24 59 33	33 29 37	42 38 36	36 52 48	73 45 40	46 20 20	44 46 56	45 46 39
	31 38 39	23 39 40	61 40 38	43 32 38	56 41 48	52 30 38	32 48 45	27 46 31
	36 46 47	34 49 49	43 39 30	47 38 47	44 40 34	45 30 32	81 76 61	98 46 28
	39 43 44	24 48 31	42 35 21	24 32 35	33 33 41	52 34 42	39 79 55	41 28 18
	1628 17 ^h 24 ^m		1920 18 ^h 00 ^m				1649 16 ^h 48 ^m	
	Wh	0.993	Sk	0.898			Si	0.839
	45 35 29	18 33 40	14 27 34	65 42 21			35 65 59	55 34 32
	31 26 13	30 39 39	24 34 43	28 31 17			41 45 53	77 48 19
	26 29 57	53 43 60	36 29 41	22 32 25			84 86 51	44 48 50
	62 34 53	64 52 58	34 41 49	44 58 37			68107101	64 42 54
	43 41 72	55 39 31	30 40 41	30 32 36			86 76 65	64 72 82
	44 34 48	76118 34	20 53 96	58 29 36			49 43 19	32 45 33

hood of missing nebulae in the rich star fields of low latitude.

When the counts have been reduced for all nine areas, it is planned to convert them to a common limiting magnitude by a carefully arranged program of sample recounting. It will then be possible to increase substantially the heights of the solutions for B by including in a single solution counts from more than one area. This program should result in significant deter-

minations of the galactic extinction for different longitude intervals.

REFERENCES

- Holmberg, Erik. 1957, *Medd. Lunds Obs.* (II) No. 136.
 Hubble, Edwin. 1934, *Ap. J.* **79**, 8.
 ——. 1936, *Ap. J.* **84**, 517.
 Humason, M. L., Mayall, N. U. and Sandage, A. R. 1956, *A. J.* **61**, 97.
 Oort, J. H. 1938, *B. A. N.* **8**, 233.
 Shane, C. D. and Wirtanen, C. A. 1954, *A. J.* **59**, 285.
 Shane, C. D. 1956, *A. J.* **61**, 292.
 Shapley, Harlow. *The Inner Metagalaxy*, 1957, p. 115.

RADIAL VELOCITY MEASUREMENTS OF SOME VISUAL DOUBLE STARS

BY OTTO STRUVE AND VELTA ZEBERGS

Berkeley Astronomical Department, University of California, Berkeley, Calif.

Received March 10, 1959

Abstract. The radial velocities of 48 visual double stars observed in 1952–1958 are listed in Table I.

Radial velocities of 48 visual double stars have been measured on 125 spectrograms obtained with the Coudé spectrograph of the 100-inch telescope of the Mount Wilson Observatory. One hundred twenty-one spectrograms were obtained between July, 1952, and June, 1954, two in 1957, and two in 1958. All spectrograms except those listed in the notes to Table I have a dispersion of 10 Å/mm.

The 20 star lines contained in Petrie's (1946)

list of wavelength standards for high-dispersion spectrograms have been measured whenever possible. Broad star lines or poor quality of spectrograms occasionally reduced the number of lines measured.

Table I gives the radial velocities. The stars are listed by their ADS numbers, the brighter component first. When the brightnesses are equal, Aitken's designation is followed. Columns 2 and 3 list the spectral types and magnitudes,

TABLE I. RADIAL VELOCITIES OF SOME VISUAL DOUBLE STARS

Star	Spectral type	Mag.	Date	UT	Qual. of lines	Radial velocity (km/sec)		Notes
						S & Z	Wilson	
ADS 683	A gF0	6.3	1953 Nov. 26	2 ^h 36 ^m	nn	+ 8.3 ± 1.6	+ 5.0	65 Psc
	B gF0	6.3		3 20	nn	+ 4.6 1.6	+ 6.8	
903	A dF5	6.8	Nov. 26	5 11	s	- 8.6 0.6	- 7.4	77 Psc
	B dF4	7.6		4 17	s	- 6.9 0.4	- 10	
3353	A dF2	7.2	Nov. 25	10 00	s	+ 3.7 0.5	+ 4	
	B dF3	7.2		8 55	n	+ 15.9 1.7	+ 3.5	
4849	A dF4	6.7	Nov. 26	11 44	s	+ 5.4 0.7	+ 6.8	
	B dF4	7.5		10 33	s	+ 7.4 0.4	+ 9	
5166	A dF6	7.2	Nov. 25	12 45	s	+ 3.1 0.5	+ 0.4	20 Gem
	B dF6	8.1		11 26	nn	+ 5.0 2.5	+ 2	
6483	AB dF6	6.4	Jan. 28	7 28	s	+ 4.7 0.3	- 0.2	
6977	A dF5	6.7	Mar. 27	3 15	s	- 12.6 0.8	- 18.2	
	B dF6	7.5		7 03	s	- 13.9 0.6	- 17.8	
6988	A dG6	4.2	Nov. 26	11 58	s	+ 16.5 0.4	+ 16.0	♄ Cnc
7187	A dF3	6.8	Mar. 31	7 08	s	+ 29.6 0.4	+ 29.1	
	B dF4	7.2		5 35	s	+ 29.0 0.4	+ 32.3	
7307	A dF3	6.5	1954 Mar. 15	6 14	s	- 4.9 0.4	+ 0.6	
	B dF2	6.8		6 59	s	- 4.8 0.3	- 1.9	
8119	A dGo	4.4	1953 Jan. 28	8 40	s	- 18.3 0.3	- 15.5	♁ UMa
	B dGo	4.9		9 54	s	- 14.2 0.3	- 15.9	
8148	A dF4	4.0	Jan. 30	10 32	s	- 15.3 0.4	- 10.3	♄ Leo
8202	A dF6	5.8	1954 May 15	3 52	s	+ 6.9 0.6	+ 4	17 Crt
	B dF7	5.9		4 59	s	+ 7.2 0.4	+ 9.8	
8257	A (F5)	(7.05)	Mar. 14	7 08	nn	+ 2 8	—	
	B (F5)	(8.9)		9 06	s	+ 4.7 0.5	—	
8406	A dA8	5.8	Mar. 13	10 31	n	+ 7.6 1.4	+ 4.8	2 Com
	B dF2	7.5		8 33	s	+ 8.6 0.7	+ 12.3	
8505	A dF4	6.6	Mar. 16	6 00	s	- 0.7 0.5	- 1	
	B dF5	7.0		7 14	s	- 1.4 0.4	+ 0.6	
8519	B dF2	7.1	1953 Apr. 1	7 23	n	- 19.1 0.8	- 15	
8561	A dF8	7.4	1954 Mar. 15	13 03	s	- 2.0 0.7	- 0.1	
	B dG2	8.0		12 08	s	- 2.5 0.8	+ 1.3	
8627	A dF6	6.0	1952 July 7	4 44	s	- 10.2 0.5	- 11.1	
8714	A (F2p)	(8.0)	1954 Mar. 15	10 56	n	+ 9 2	—	
	B —	—		8 58	s	+ 4.3 1.0	—	
8786	A dF5	7.2	Mar. 16	10 45	s	- 88.3 0.6	- 92.2	
	B —	—		6 31	s	- 90.7 0.7	—	
	B dGo	7.6	1957 Apr. 9	10 16	s	- 91.2 0.5	—	
			1958 Mar. 4	9 11	s	- 89.2 0.7	- 86.5	
8883	A (G5)	(7.35)	1954 May 20	12 17	s	- 90.0 0.4	—	
	B (G5)	(7.65)		8 45	s	- 90.4 0.6	—	
9053	A dF7	6.5	May 19	4 15	s	+ 27.9 0.6	—	
	B dG1	7.7		4 46	s	+ 27.8 0.7	—	
9174	AB Fo+A2	6.8	1953 June 3	4 40	n	- 13.9 1.5	- 19.0	54 Hya
	B dF9	7.1		5 51	s	- 15.3 0.4	- 19.4	
9375	A dG5	4.8	1954 May 18	3 48	n	- 7.2 1.0	- 8.3	♁ Boo
	B dG5	4.8		4 56	s	- 19.5 1.0	- 19.9	
9413	A dK5	6.8	May 17	4 52	s	+ 2.8 0.4	+ 3.9	
	B Fo	7.1		5 03	n	+ 1.7 0.5	+ 6	
9493	A dG5	7.3	1953 Mar. 27	4 04	n	+ 3.5 0.7	+ 8.4	
	B dG5	7.4		8 44	n	- 8.4 1.2	- 8.4	
9507	A dG5	6.8	Apr. 1	9 45	s	- 2.6 1.4	—	
	B dG5	7.6		12 11	s	- 33.2 0.5	- 34.5	
9535	A dG5	6.8	1954 Mar. 14	11 06	s	- 37.2 0.6	- 34.2	
	B dG6	6.7		8 23	s	- 39.6 0.5	- 36.7	
9580	A dF5	6.7	1953 Mar. 31	9 30	s	- 39.8 0.7	- 40.1	
	B —	—		13 15	s	- 38.4 0.7	- 45.5	
	B —	—	May 17	5 46	s	- 38.9 0.8	—	
				12 02	s	- 35.2 0.5	—	
9617	A dF9	5.0	May 17	6 49	s	- 36.1 0.5	—	
	B —	—		4 12	s	- 9.1 0.4	- 6.8	
	AB —	—	May 21	4 42	s	- 8.0 0.6	—	η CrB
				4 47	s	- 7.9 0.5	—	
9728	A dF6	6.5	1952 July 7	4 35	s	- 5.9 1.5	—	
	B —	—		4 52	s	- 8.3 0.5	—	
			July 9	4 58	s	- 7.5 0.4	—	
				5 50	s	- 8.6 0.4	0	
			July 10	4 24	s	- 7.8 0.4	—	
				5 21	s	- 7.7 0.5	—	
				4 45	s	- 7.3 0.6	—	

TABLE I (continued)

Star	Spectral type	Mag.	Date	UT	Qual. of lines	Radial velocity (km/sec)		Wilson	Notes
						S & Z			
B	dF6	6.6	1953 Apr. 2	9 38		-1.5	0.8		
			Apr. 3	9 31		-1.0	0.4		
			June 3	4 59		+1.4	0.6		
			1952 July 7	6 54	s	+2.0	0.5	+3.5	
			July 9	4 55		+1.4	0.4		
			July 10	4 19		+0.9	0.6		
				5 10		+1.8	0.5		
			1953 Apr. 2	8 50		+1.4	1.0		
			Apr. 3	8 30		+2.2	0.4		
			June 3	4 48		-0.2	0.4		
9969 A	dKo	7.5	1954 Mar. 16	11 48	s	+19.1	0.6	+18.1	49 Ser
B	dK1	7.6		12 50	s	+20.0	0.7	+20.8	
10157 A	dGo	3.0	1953 Aug. 19	4 15	s	-67.4	0.5	-69.9	‡ Her
				4 34		-67.8	0.6		
				4 41		-68.8	0.4		
			Aug. 22	3 42		-67.7	0.4		
				4 22		-68.4	0.3		
B	—	—	Aug. 19	3 52	s	-73.0	1.1	—	
			Aug. 22	4 22		-71.1	0.5		
				4 53		-70.4	0.4		
10993 B	gG3	5.2	Apr. 3	11 45	s	-33.0	0.6	-31.0	95 Her
11483 A	dGo	6.8	1952 July 8	7 35	s	+10.4	0.5	+9.6	
			July 9	9 42		+9.0	0.5		
B	dF8	7.2	July 8	8 36	s	+7.6	0.5	+2.1	
11639 A	dA9	4.3	1953 Aug. 19	6 05	s	+24.9	0.9	-26.0	‡ Lyr
12145 A	dG4	8.2	1952 July 9	8 38	s	+22.2	0.6	+28	
			1953 Aug. 20	5 12		+15.8	2.8		
BC	dKo	8.7	1952 July 9	7 30	s	+26.1	0.5	+24.8	
			1953 Aug. 20	4 00		+25.2	0.8		
12169 A	dG3	6.6	Aug. 20	7 07	s	-40.5	1.5	-37.6	
			Aug. 22	7 14		-40.9	0.9		
B	dG5	6.8	Aug. 20	6 18	s	-43.5	0.9	-41.1	
			Aug. 22	6 13		-42.4	0.7		
13868 A	dF6	6.6	Aug. 19	8 33	s	-12.6	0.5	-16.2	
B	—	—		7 11	s	-15.2	0.3	—	
14270 A	dG9	7.5	1952 July 10	9 58	s	-29.7	0.7	-25.6	
B	dG8	8.2		10 58	s	-31.6	0.7	-29.1	
14279 A	sgK1	4.5	1954 June 17	11 02	s	-7.2	0.8	-6.6	γ Del
B	dF6	5.5	June 18	11 00	s	-6.2	0.7	-7.6	
14636 A	dK6	5.6	1953 Aug. 19	11 39	s	-64.8	0.6	-64.3	61 Cyg
			Aug. 20	8 49		-64.2	1.1		
B	dMo	6.3	Aug. 19	10 07	s	-64.4	0.6	-63.5	
			Aug. 20	8 04		-65.5	1.6		
15971 A	dF2	4.4	Aug. 20	9 45	nn	+20.8	2.7	+24.9	‡ Aqr
B	dF1	4.6		10 52	n	+31.8	1.7	+28.9	
16417	dG1	6.5	1952 Dec. 7	4 14	s	-20.0	0.9	-26.5	
				5 02		-22.8	1.4		
16611 A	dF5	8.3	1953 Aug. 22	11 23	s	-29.4	0.7	-31.8	
B	dG2	9.7		9 55	s	-29.4	1.2	-24	
16979 A	A5	5.4	1952 July 9	11 38	nn	+10	4	-1.8	107 Aqr
B	—	—	July 8	11 32	n	-4.2	1.8	—	
17149 A	dGo	6.6	1953 Aug. 21	9 57	s	-7.3	0.4	-4.6	
			Aug. 22	8 46		-6.9	0.6		
B	dGo	6.6		8 13	s	-8.4	0.6	-7.7	

NOTES

- 3353 B Spectroscopic binary. Poor spectrogram.
 8119 A Spectroscopic binary. 5 Å/mm dispersion.
 B Spectroscopic binary. 5 Å/mm dispersion.
 8148 A 5 Å/mm dispersion.
 8202 A 5 Å/mm dispersion.
 B 5 Å/mm dispersion.
 8627 A Spectroscopic binary; orbit Sanford, Karr.
 9413 B H omitted from mean radial velocity.
 9617 Spectroscopic binary; orbit Chang (1929).
 Radial velocity of A agrees well with
 Chang's velocity curve. Radial velocity of
 B is close to the γ-velocity, perhaps slightly
 contaminated by A.
 9728 A Spectroscopic binary.
 10157 Spectroscopic binary; orbit Berman (1941).
 Radial velocity of A agrees well with Ber-

man's velocity curve. Using the mass ratio
 of $M_B/(M_A + M_B) = 0.39$, given by Ber-
 man, the radial velocity of B should be
 -72.6 km/sec; our observed value is -71.5
 km/sec. The spectrogram of A taken at
 3:42 UT, Aug. 22, has a dispersion of 2.8
 Å/mm.

- 10993 B 5 Å/mm dispersion.
 11639 A Spectroscopic binary; orbit Jordan.
 12145 Spectrograms taken on August 20 are of
 poor quality, dark background. Wilson
 lists B, presumably the same as Aitken's
 BC.
 14636 Spectrograms taken on August 20 are of
 poor quality, underexposed.
 16147 Both spectrograms underexposed.
 16611 B Spectroscopic binary.

respectively, from *The General Catalogue of Stellar Radial Velocities* by R. E. Wilson. For stars not listed by Wilson the spectral types and magnitudes from Aitken's double star catalogue are given in parentheses whenever available.

Three classifications from visual estimates are given for the quality of the absorption lines: s denotes sharp lines, n broad, nn very broad.

The velocities measured by us are listed together with the mean error for each spectrogram. No systematic corrections have been made, and the mean errors are only an indication of the internal consistency of measurements for each spectrogram, reflecting the quality of star lines and the quality of the spectrogram. The real accuracy of the radial velocities is probably considerably better than is indicated by the mean errors in the table. The latter are influenced to some extent by blends and inaccuracies of the adopted stellar wave length. The values $v_A - v_B$ should be especially accurate because in nearly all cases the two components of each double star were observed in immediate succession and with precisely the same adjustments of the spectrograph.

Table I also gives the radial velocities from Wilson's catalogue. The differences between our measurements and the catalogue values were investigated, excluding all known spectroscopic binaries and measured values of low weight. Means were used when more than one spectrogram of each star had been measured. The mean difference $v_{s\&z} - v_{\text{Wilson}} = 0.0$ km/sec.

The last column of Table I gives the Greek letter or Flamsteed designation.

The average difference $|v_A - v_B|$ for all well-observed binaries, 29 in number, is 1.9 km/sec. Since the uncertainty of measurements given by

the mean plate errors is approximately ± 1.0 km/sec, we estimate that the difference $1.9 - 1.0$, or about 1 km/sec, represents the average radial component of the relative orbital motion of the pairs included in this study. An attempt to group the values of $|v_A - v_B|$ according to the observed separations between the components corrected for their differences in distance from the sun gave the following results:

Separation in a.u.	Mean $ v_A - v_B $	$\sqrt{\frac{(v_A - v_B)^2}{n}}$
0-250	2.3 km/sec	3.4 km/sec
250-2000	1.3	1.7

Only main sequence stars have been included in this comparison and their absolute magnitudes were taken from the H-R diagram as given by Allen (1955). The observed separations were corrected by the factor $(I_{\text{abs.}}/I_{\text{app.}})^{1/2}$, and the angular separations converted to astronomical units. There is perhaps a slight indication of correlation between the separations and the values of $|v_A - v_B|$ or the root mean square differences of velocity, but the amount of material is not sufficient to regard this as established.

We express our thanks to Messrs. A. Weil and T. Henyey for many radial velocity measurements and reductions. A helpful suggestion by the referee has been included in the discussion. The spectrograms used in this investigation were obtained by Struve as guest investigator at the Mt. Wilson Observatory.

REFERENCES

- Allen, C. W. 1955, *Astrophysical Quantities* (The Athlone Press: University of London), p. 177.
 Berman, L. 1941, *Pub. A. S. P.* **53**, 22.
 Chang, Y. C. 1929, *Ap. J.* **70**, 182.
 Petrie, R. M. 1946, *Contr. Dom. Ap. Obs. Victoria* **1**, 43.

THE SHORT-PERIOD VARIABLE, SX PHOENICIS

By FRANK BRADSHAW WOOD

Flower and Cook Observatory, University of Pennsylvania, Philadelphia, Pa.

Received April 16, 1959

Abstract. Two- and three-color observations of SX Phoenicis are described, and the possible existence of occasional rapid fluctuations is noted.

Jackson (1938, 1949) first called attention to the large proper motion of SX Phoenicis (HD 223065, CoD-42° 16457, CPD-42° 9607, GC 32998). The variation of light was announced by

Eggen (1952). The star is an intrinsic variable with period approximately 80 minutes and range of light variation which has been observed to be as small as 0.3 mag. or as large as 0.8 mag.

From the trigonometric parallax and apparent magnitude Eggen computed the absolute magnitude to be +4.3; this falls within the range of possible values discussed by Jackson. The large proper motion and the high galactic latitude (-71°) also indicate that the star is relatively near and intrinsically faint. The fundamental data have been given by Evans, Menzies, and Stoy (1957). Kuiper (1940) described the star as a "probable subdwarf" (A3); Joy (1947) classified the spectrum as sd A2s. Walraven (1953, 1955) observed the star on 17 nights in one wavelength band and interpreted the light curve in terms of a beat-period phenomenon. D. Wilson and Walker (1956) observed the system spectroscopically and photoelectrically. On only one night were the photometric observations

considered satisfactory; the radial velocity curve did not show the large variations in range found in the light curve.

The present study is based upon observations on three nights in 1957; on two of these the system was observed in three wavelength bands and on the other in two. The material is not sufficient in quantity to extend Walraven's analysis, but it is useful for a description of the light variation in three colors at this epoch. In addition to my own observations, I was able to study tracings made by A. H. Hogg and K. Gottlieb of the Mount Stromlo Observatory. These give an epoch of maximum of JD 243 5689.1181.

The observations are listed in Table I. Magnitude difference is in the natural system of the photometer. The observations were made with

TABLE I. PHOTOELECTRIC OBSERVATION OF SX PHE

Yellow									
JD (hel) 2436 +	Comp. -Var.	JD (hel) 2436 +	Comp. -Var.	JD (hel) 2436 +	Comp. -Var.	JD (hel) 2436 +	Comp. -Var.	JD (hel) 2436 +	Comp. -Var.
158.9507	+1 ^m 401	175.9631	+1 ^m 664	176.0149	+1 ^m 511	183.9798	+1 ^m 407	184.0198	+1 ^m 372
.9548	1.335	.9637	1.679	.0195	1.704	.9898	1.988	.0306	1.346
.9651	1.393	.9697	1.708	.0202	1.724	.9905	2.013	.0326	1.436
.9675	1.418	.9782	1.599	.0244	1.863	.9991	1.736	.0393	1.649
.9725	1.558	.9823	1.509	.0286	1.823			.0451	1.817
.9823	1.763	.9862	1.479	.0454	1.392			.0461	1.802
.9860	1.726	.9968	1.309	.0507	1.355				
				.0554	1.317				
Blue									
JD (hel) 2436 +	Comp. -Var.	JD (hel) 2436 +	Comp. -Var.	JD (hel) 2436 +	Comp. -Var.	JD (hel) 2436 +	Comp. -Var.	JD (hel) 2436 +	Comp. -Var.
158.9520	+1 ^m 720	175.9658	+2 ^m 143	176.0068	+1 ^m 700	183.9823	+1 ^m 871	184.0010	+2 ^m 032
.95569	1.652	.9664	2.152	.0098	1.799	.9831	1.911	.0017	2.014
.95583	1.628	.9709	2.113	.0109	1.819	.9839	1.951	.0216	1.691
.95597	1.602	.9718	2.088	.0163	1.974	.9929	2.434	.0341	1.852
.95611	1.645	.9735	2.123	.0169	2.009	.9941	2.377	.0350	1.881
.95625	1.632	.9802	1.982	.0221	2.225			.0417	2.066
.95639	1.656	.9808	1.972	.0260	2.348			.0424	2.090
.95653	1.667	.9834	1.923	.0264	2.325				
.95667	1.674	.9890	1.820	.0310	2.182				
.95681	1.674	.9883	1.719	.0476	1.804				
.95694	1.656			.0521	1.752				
.95708	1.632			.0564	1.713				
.95722	1.658			.0575	1.745				
.95736	1.674								
.95750	1.683								
.9665	1.762								
.9696	1.812								
.9703	1.836								
.9710	1.869								
.9738	1.952								
.9746	1.993								
.9841	2.212								
.9902	2.043								
Ultraviolet									
JD (hel) 2436 +	Comp. -Var.	JD (hel) 2436 +	Comp. -Var.	JD (hel) 2436 +	Comp. -Var.	JD (hel) 2436 +	Comp. -Var.	JD (hel) 2436 +	Comp. -Var.
175.9682	+2 ^m 274	176.0125	+2 ^m 091	183.9862	+2 ^m 332	184.0032	+2 ^m 063		
.9745	2.223	.0180	2.239	.9868	2.387	.0038	2.033		
.9761	2.205	.0187	2.256	.9874	2.436	.0274	1.919		
.9813	2.099	.0233	2.333	.9971	2.275	.0366	2.072		
.9849	2.073	.0273	2.397						
.9926	2.013	.0334	2.184						
		.0489	1.955						
		.0534	1.892						
		.0586	1.876						

the 20-inch Catts reflector of the Mount Stromlo Observatory and an EMI type 5060 multiplier photocell. The mechanical and optical parts of the photometer were conventional. The amplifier was constructed by Dr. W. M. Protheroe; a description will soon be published. Although it combines both direct-recording and integrating features, the rapidity of the light changes was such that only the direct-recording features were used. Glass color filters were ultraviolet Jena UG 2, 2mm (Schott); blue GE 13, 2mm +BG 12, 1mm (Schott); and yellow, Chance OY 4. The relations between this color system and the *U, B, V* system will shortly be published by B. Westerlund.

The comparison star was CoD-42° 16461, HD 223107 (G5). Its magnitude and color changes were linear with sec *z*; there is no reason to suspect it of variability.

The method of observing was to measure comparison star and sky in three wavelength bands (on the first night in two) and to observe the variable in each of the three alternately. Observations of the variable were then continued in alternate bands with frequent checks of the sky background. Normally, a measure in each band occupied one minute, but when rapid changes were occurring the star was occasionally followed continuously for as long as five minutes. After thus observing the variable for intervals which normally ran for twenty minutes the comparison star was again measured. In making the reductions, the intensity of the comparison star was found by interpolation to the time of each observation of the variable. When observation in one wavelength was carried on continuously for sev-

eral minutes or showed appreciable change during the measure, the tracing was separated into two or more equal time intervals and each of these was reduced separately.

Periods. Dr. Th. Walraven has compared the observations with the ephemeris previously given by him (Walraven 1953); the results are shown in Table II. The first three columns give the observed maxima, the moments of mean light on the ascending branch computed by formula (2) in Walraven's paper, and the difference between these. The variation of the (*O*−*C*) was compared with that shown in Fig. 1 of his paper: a satisfactory agreement is obtained if the curve there shown is shifted to the left by 0.1 of the phase ψ . The fourth column lists the computed phase using the observed maxima and Walraven's formula (3). The values of $\Delta\phi$ in the fifth column are read from the curve using these phases corrected by 0.1. In the sixth column, these are reduced to days and in the seventh the residuals found by subtracting the sixth column from the third.

Walraven points out that in these observations maximum occurs 0^h0207 later than the ascending branches according to the 1952 observations, indicating the primary period has lengthened. By 1957 the beat phenomenon had shifted in phase so that the variations in phase and amplitude arrive about 0.1 beat-period earlier than in 1952. Thus the beat-period has shortened and this suggests that the period, *P*₀, has increased relatively more than the period *P*₁.

The light curve. The essential data of the light curve on each of the three nights are assembled in Table III. These were determined from study

TABLE II. PHASE OBSERVATIONS

Max. obs. JD 2436 +	Asc. br. comp.	<i>O</i> ₁ − <i>C</i> days	ψ comp.	$\Delta\phi$ phase	$\Delta\phi$ days	Corr. <i>O</i> − <i>C</i> days
158.9819	.9630	+ .0180	.252	− .026	− .0014	.0203
175.9674	.9469	+ .0205	.334	− .005	− .0003	.0208
176.0257	.0019	+ .0238	.636	+ .038	+ .0021	.0217
183.9910	.9717	+ .0193	.942	− .020	− .0011	.0204
184.0447	.0267	+ .0180	.225	− .030	− .0016	.0196

TABLE III. AMPLITUDE OBSERVATIONS

JD 2436000 +	Yellow	Blue	Ultraviolet
158	0 ^m 44 (0 ^m 43–0 ^m 47)	0 ^m 58 (0 ^m 58–0 ^m 64)	
175–6	0 ^m 40 (0 ^m 40–0 ^m 44) 0 ^m 56 (0 ^m 56–0 ^m 60)	0 ^m 45 (0 ^m 45–0 ^m 48) 0 ^m 64 (0 ^m 64–0 ^m 68)	0 ^m 28 (0 ^m 27–0 ^m 32) 0 ^m 40 (0 ^m 39–0 ^m 44) (P) 0 ^m 52 (0 ^m 52–0 ^m 56) (F)
183–4	0 ^m 70 (0 ^m 68–0 ^m 72) 0 ^m 50 (0 ^m 48–0 ^m 54)	0 ^m 78 (0 ^m 75–0 ^m 79)	0 ^m 60 (0 ^m 55–0 ^m 64)

(P) as measured from preceding min.
(F) as measured from following min.

f tracings, as well as plots of the listed observations. Since it was not possible to observe simultaneously in all colors, this method assumes that the maxima occur simultaneously in all colors. There is no evidence to the contrary but, because of the rapidity of the light changes, it would be difficult to detect any lag unless it reached at least two minutes ($P_0/40$).

By each of the ranges are figures giving the estimated minimum and maximum values permitted by the observations. The range of light variation in blue was larger than in the yellow although not always by the same amount. The range in ultraviolet was the smallest of the three. In yellow and blue, the variation in range seems to be caused by fluctuations in the heights of maximum; minimum light (with possibly one exception) remains constant from cycle to cycle. In the ultraviolet, however, the brightness at minimum varied by as much as 0.12 mag.

From the published observations of Wilson and Walker similar ranges were computed. The larger range in yellow compared to ultraviolet is not shown in these observations. These were made at unavoidably large zenith distances, with sec z ranging up to 6.5. No comparison star was used because the photometric observations were made simultaneously with the spectrographic ones. Because of these factors, it is not possible to conclude definitely that the system behaves differently in this respect at different epochs, although it seems likely that this is the case. The magnitudes at three minima reported by Wilson and Walker do not repeat precisely in the ultraviolet but do so in the longer wavelengths within normal observational error; this agrees with the evidence discussed in the preceding paragraph.

It is difficult to derive a color curve in the usual manner, because of the rapidity of the color changes. The colors listed in Table IV were observed on December 3, 1957 when the longest uninterrupted run was made. They were computed by interpolating the yellow and ultraviolet observations to the times of the blue, using only cases where inspection of the light curve indicated the light changes were linear. In a few cases the observations were extrapolated to the time of the blue if they fell within two minutes of it. The colors were then reduced to outside the atmosphere using coefficients determined from observations of the companion star. Second-order effects caused by color differences of comparison and variable were ignored. The maximum zenith

TABLE IV. OBSERVED COLORS

Phase	B-Y	UV-B	Phase	B-Y	UV-B
0 ^d 0551	+ ^m 667	-1.446	0 ^d 0495	+ ^m 670	-1.422
.0035	+ .677	-1.422	.0547	+ .708	-1.528
.0131	+ .642	-1.433	.0000	+ .714	-1.526
.0160	+ .657	-1.438	.0050	+ .642	-1.508
.0216	+ .668	-1.357	.0219	+ .624	-1.389
.0489	+ .662	-1.404	.0264	+ .690	-1.362

distance for which colors were computed was 48°. The phases in Table III are from the preceding maximum. A plot of $B-Y$ against $UV-B$ was made to search for evidence of a loop-wise progression with phase such as found by Hardie and Geilker (1958) in RR Lyrae and by Tift and Smith (1958) in T Sextantis. Color changes from cycle to cycle make it impossible to confirm the existence of such a loop on the basis of the present evidence.

Finally, Figure 1 reproduces a portion of the recordings taken on the first night of observation. The tracing showing the fluctuation in the blue intensity between 8^h54^m0 and 8^h56^m8 Stromlo time is difficult to interpret. The natural suspicion is to blame this on varying transparency in the earth's atmosphere. However, fluctuations caused by poor sky usually (although not always) give more erratic traces than this; moreover, when changes of this amplitude are seen on the tracings, it has been my experience that careful

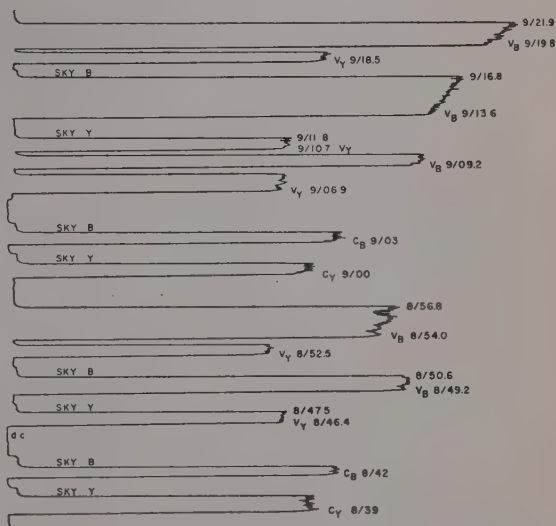


Figure 1. Possible fluctuations in SX Phe, blue light. The fluctuations between 8/54.0 and 8/56.8 are difficult to interpret. In the blue observation following the last shown, the pen was pinned at the top of the chart; reduced gain was needed to make the measure.

inspection of the sky shows signs of trouble; such was not the case. The star was carefully centered in the diaphragm at the beginning of the observation and was still centered at the end; no trouble with the telescope drive occurred during the night. Nevertheless, it must be granted that in the case of a normal star, such a tracing probably would not even be reduced. It is shown here partly because the changes are at the same rate as those shown 20 minutes later when the light was increasing most rapidly. It may be pertinent to note that the star averaged 0.1 mag. fainter at this time than at the other minima in this color. Certainly further observations should pay particular attention to the star at this part of the curve. In the listed observations, the tracing was subdivided into time intervals of 0.2 minute and the relative difference of magnitude was computed for each.

As in so many cases, the chief conclusion is that further observations are needed. It is important to observe in the ultraviolet, as well as in other colors; simultaneous observation with two or more telescopes would be particularly helpful.

I am indebted to Drs. Th. Walraven and H. J. Smith for criticism of the first draft typescript and to Drs. A. R. Hogg and B. Westerlund for sending information concerning the equipment. I am especially grateful to the staff at Mount Stromlo for yielding telescope time and for efficient aid in installing the photometer and keeping the telescope in operation. The work was made possible by the combination of a sabbatical leave and a Fulbright award. The electronic parts of the photometer were constructed by Dr. W. M. Protheroe with the aid of a special research grant from the University of Pennsylvania.

REFERENCES

- Eggen, O. J. 1952, *Pub. A. S. P.* **64**, 31, 305.
 Evans, D. S., Menzies, A., and Stoy, R. H. 1957, *M. N.* **117**, 534.
 Hardie, R. H. and Geilker, C. D. 1958, *Ap. J.* **127**, 606.
 Jackson, J. 1938, *M. N.* **98**, 506.
 —. 1949, *M. N. Astr. Soc. South Africa* **8**, 29.
 Joy, A. H. 1947, *Ap. J.* **105**, 96.
 Kuiper, G. P. 1940, *Ap. J.* **91**, 269.
 Tift, W. G. and Smith, H. J. 1958, *Ap. J.* **127**, 591.
 Walraven, Th. 1953, *B. A. N.* **12**, 57.
 —. 1955, *ibid.*, p. 223.
 Wilson, O. C. and Walker, M. F. 1956, *Ap. J.* **124**, 325.

SOME PERIODIC ORBITS IN THE RESTRICTED PROBLEM OF THREE BODIES AND THEIR STABILITIES

By P. J. MESSAGE

Yale Observatory, Yale University, New Haven, Conn.*

Received March 17, 1959

Abstract. Periodic orbits in the restricted problem of three bodies near the exterior case of 2:1 commensurability of period have been found by step-by-step numerical integration, using the IBM 650 computer at the Yale University Computing Center. Periodic orbits with eccentricities up to 0.4 have been traced on the two series of symmetric orbits and also on one of the series of asymmetric orbits, and on one series of symmetric orbits members were traced with eccentricities up to 0.9. For the orbits of all but the largest eccentricities the coordinates are exhibited as Fourier series. The ordinary stabilities of a selection of the orbits of eccentricities up to 0.1 were investigated using a method deriving from Brown's method for the determination of the motion of the lunar perigee. The symmetric orbits of small eccentricity prove to be stable, those of larger eccentricity unstable, and the asymmetric orbits investigated are stable, there being an exchange of stability at the point of bifurcation.

In a paper (Message 1958) presented to the Symposium on Celestial Mechanics held in New York in March 1958 I described how an investigation based on the secular and critical terms of the disturbing function in cases of near-commensurability of period led to the conclusion that there are, in certain of the exterior cases of commensurability, periodic solutions of the restricted problem of three bodies which are asymmetric, as well as those which are symmetric and which

have been studied by many writers (Poincaré 1892; Darwin 1898, 1909, 1912; Hill 1902; Schwarzschild 1903; van Veen 1927). I also described the beginning of a project to illustrate these periodic orbits and investigate their properties further by tracing examples by step-by-step numerical integration, using the IBM 650 computer of the Yale University Computing

* Now at Gonville and Caius College, Cambridge, England.

enter. The aim of the present paper is to describe some periodic orbits which have been traced, belonging to three series of orbits in the exterior commensurability case, two consisting of symmetric orbits and the third of asymmetric, and to investigate the stability of a selection of these orbits.

In the restricted problem of three bodies as usually defined, the system considered consists of a central body S of mass M , a body J of mass m' moving about S in a circular orbit of radius r_s , and a body P of negligible mass moving in the field of both. The system is referred to rectangular axes with origin O at the center of mass of S and J , the axes rotating about the normal to the orbit plane of J with angular velocity n' , the angular velocity of J in its orbit, so that the axis OX may be chosen so that S and J are both always on it, their coordinates being $\{-\alpha'm'/(1+m'), 0, 0\}$ and $\{\alpha'/(1+m'), 0, 0\}$, respectively. The coordinates (x, y, z) of P are governed by the equations of motion

$$\begin{aligned} \frac{d^2x}{dt^2} - 2n' \frac{dy}{dt} - n'^2 x &= -\frac{\mu}{r_s^3} \left(x + \frac{\alpha'm'}{1+m'} \right) - \frac{\mu m'}{r_j^3} \left(x - \frac{\alpha'}{1+m'} \right) \\ \frac{d^2y}{dt^2} + 2n' \frac{dx}{dt} - n'^2 y &= -\frac{\mu}{r_s^3} y - \frac{\mu m'}{r_j^3} y \\ \frac{d^2z}{dt^2} &= -\frac{\mu z}{r_s^3} - \frac{\mu m' z}{r_j^3}, \end{aligned} \quad (1)$$

where

$$\begin{aligned} \mu &= GM, \\ r_s^2 &= \left(x + \frac{\alpha'm'}{1+m'} \right)^2 + y^2, \\ r_j^2 &= \left(x - \frac{\alpha'}{1+m'} \right)^2 + y^2, \end{aligned}$$

G being the constant of gravitation. The orbits traced all lie in the orbit plane of J , and so z is always zero. The first two equations were integrated by the central difference method described in the previous paper.

A periodic orbit, in the sense usual in this context, is one in which the perturbations in the orbital elements are periodic functions of the time, the period (T , say) being equal to the synodic period of P and J . Since the coordinates of P are referred to a frame rotating with J , they too are periodic functions with this period. In the commensurability case at present under dis-

cussion the mean motions n and n' of P and J , respectively, are related by

$$2n - n' = \sigma n, \quad (2)$$

where $|\sigma|$ is small compared to unity. Thus the orbit of P lies outside that of J , and in the rotating frame the mean motion of P is retrograde. Consequently near conjunction of P with J the coordinate y will pass from positive to negative values. The integration of each orbit was begun at the point where the mean longitudes λ and λ' of P and J , respectively, are equal. The data supplied to the computer included the osculating values of the eccentricity (e_0) and the ratio of the semi-major axes of the orbits of P and J (α_0/α'), the initial value of the mean anomaly ($\lambda_0 - \omega_0$), and the size of the step of integration. The value of the mass ratio m' employed was 0.000 954 927, as used by K. Schwarzschild (1903) for the mass of Jupiter. The program was designed to stop the integration a few steps after y began to take negative values for the second time. Interpolation by Neville's process (1934) was carried out to find the values of x , dx/dt , dy/dt , and λ' corresponding to the point in the orbit where y regains its initial value. If the orbit is precisely periodic x , dx/dt , and dy/dt also will have returned to their initial values. A search process of trial integrations followed by interpolation of residuals was carried out to find sets of data leading to orbits in which x/α' , y/α' , $(dx/dt)/\alpha'n'$ and $(dy/dt)/\alpha'n'$ simultaneously return to their original values to within five decimal places. The symmetric orbits meet the axis $y = 0$ at right angles at the end of each half period, and so the search for an orbit of this type only requires the integration of the first half of the orbit, periodic orbits being identified by this property. The accuracy of the integration was checked by evaluating the constant k in Jacobi's integral

$$\begin{aligned} \left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 - n'^2(x^2 + y^2) \\ - \frac{2\mu}{r_s} - \frac{2\mu m'}{r_j} = k, \end{aligned} \quad (3)$$

using the coordinates and velocities at different points on the orbit, and the step of integration was chosen to maintain k constant to six significant figures throughout the integration, some of the orbits with large eccentricities being integrated in more than one stage with different sized steps.

The investigation of the secular and critical terms near this commensurability, which considered eccentricities up to 0.15, showed the existence of four series of periodic orbits, two of them of symmetric orbits and two of asymmetric. In all of them the critical argument

$$\theta = 2\lambda - \lambda' - \varpi$$

is constant, apart from terms of short period. In one of the series of symmetric orbits this constant value of θ is 0, in the other it is π , and there is a member of each of these two series corresponding to each value of the mean eccentricity. The relation between α/α' and the mean eccentricity on each series derived from these terms was used to give approximate osculating values of α/α' for each osculating value (e_0) of the eccentricity at conjunction, from which the search process was begun. For higher values of e_0 on each series the search was commenced with values of α/α' extrapolated from those found for smaller e_0 . Members of the series $\theta = \pi$ were traced with e_0 ranging from 0.0125 to 0.90. Some of these are shown in the accompanying diagrams, points corresponding to equally spaced instants of time being represented by filled circles. On the orbits with $e_0 = 0.40$ and larger the angular velocity, relative to a fixed frame, at pericenter is greater than n' , and so relative to the rotating frame P has a direct motion, so that in this frame the orbit has a loop, which is larger for greater values of the eccentricity. (In the orbit with $e_0 = 0.40$, alternate points on the loop have been omitted from the diagram for clarity.) Members of the series $\theta = 0$ were traced with e_0 ranging from 0.0125 to 0.40, some of them being shown on the accompanying diagrams, points on these orbits being represented by crosses. (In the orbit with $e_0 = 0.40$, two out of every three points on the loop are omitted for clarity.)

The two series of asymmetric orbits branch from the series $\theta = \pi$ at $e = 0.0367$; on one series θ is smaller for larger values of e , and on the other it is larger, the two values of θ for a given e having the sum 2π . From this last property it can be seen by considering the expressions for the coordinates as Fourier series in the two arguments θ and $\lambda - \lambda'$ that each of the two asymmetric orbits corresponding to a given value of e is the mirror image of the other in the line $y = 0$, and so only one of these series was investigated. Members were found for values of e_0 between 0.0375 and 0.40, some of them being

shown on the diagrams, points being indicated by open circles. (In the orbit with $e_0 = 0.40$ alternate points on the loop have been omitted for clarity.) Tables I to III give, for all the orbits traced, the osculating values at conjunction of θ , e_0 , and α/α' , the period, T , in terms of that of J , the quantity σ measuring the closeness of the commensurability of period, and Jacobi's constant k .

Since the coordinates and velocities relative to the rotating frame are periodic functions of time with period T , they may be expressed as Fourier

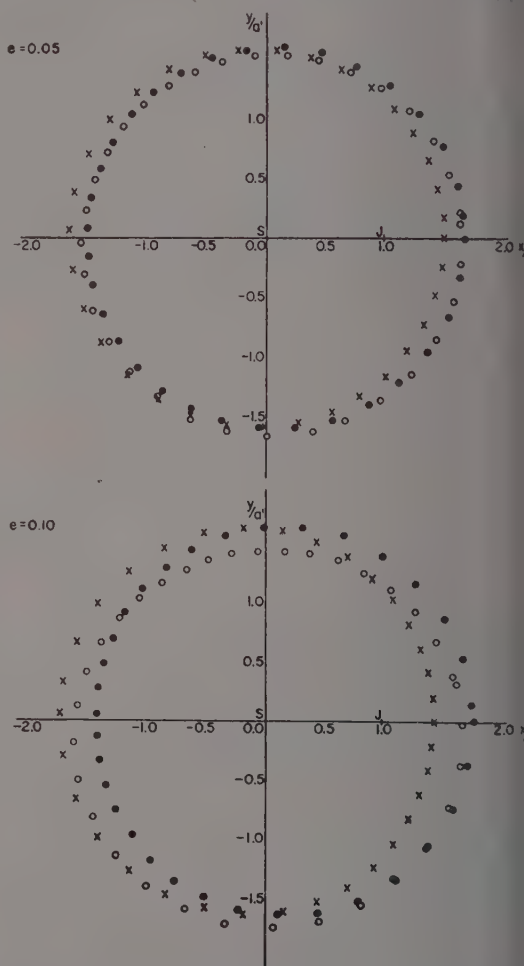


Figure 1. Points corresponding to equally spaced instants of time in periodic orbits are indicated: by filled circles for symmetric orbits of the series $\theta = \pi$, by crosses for symmetric orbits of the series $\theta = 0$, by open circles for asymmetric orbits.

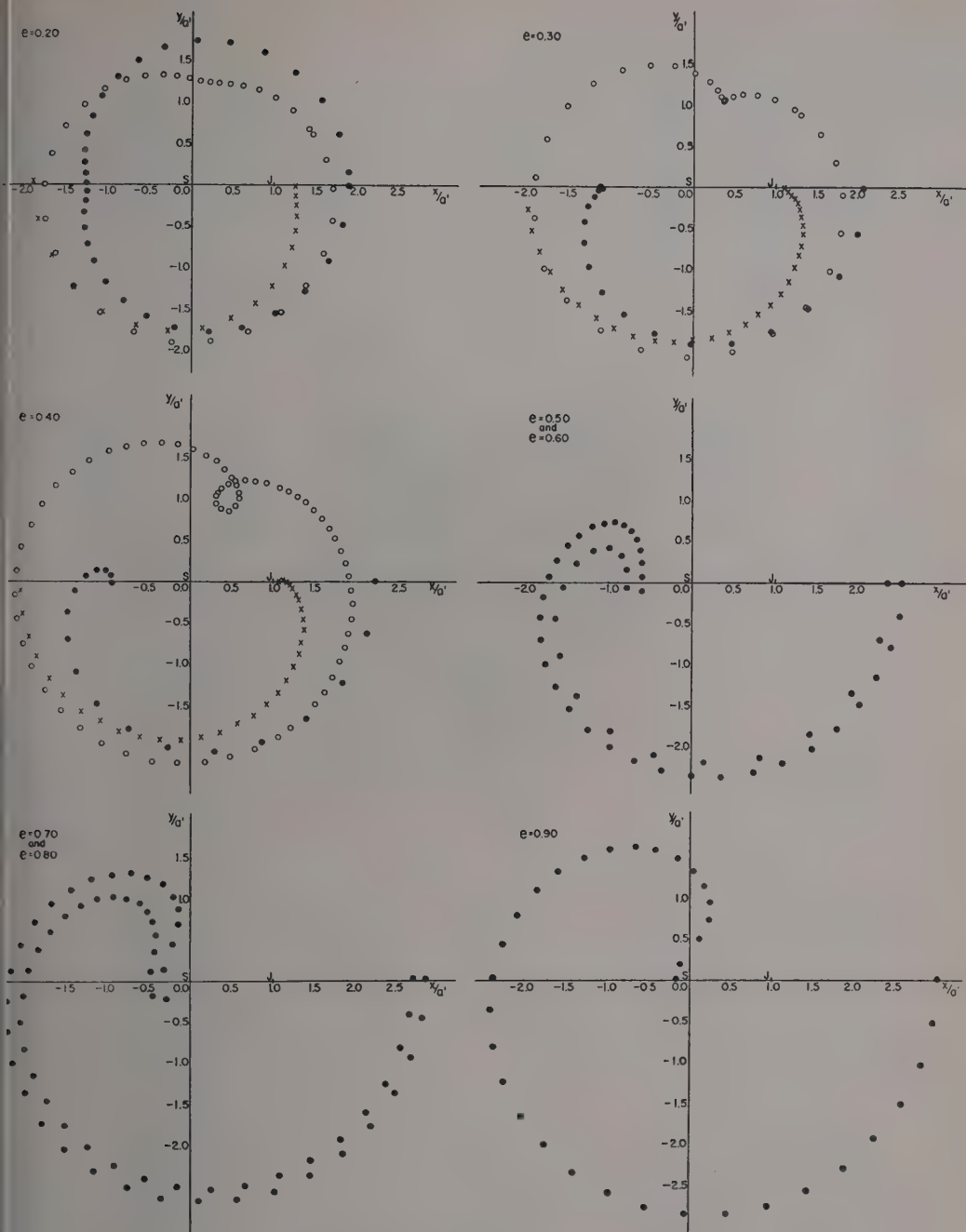


Figure 1. (continued)

TABLE I. SERIES $\theta = \pi$

e_0	a_0/a'	T/T'	σ	$k/n'^2a'^2$
0.0125	1.603293	1.980954	-0.019416	-3.15410
0.025	1.597200	1.992249	-0.007812	-3.15098
0.0375	1.595056	1.996395	-0.003618	-3.14903
0.05	1.593950	1.998587	-0.001415	-3.14709
0.10	1.592730	2.001266	0.001265	-3.13667
0.15	1.592520	2.001747	0.001744	-3.12029
0.20	1.592458	2.001722	0.001719	-3.09749
0.25	1.592386	2.001541	0.001539	-3.06801
0.30	1.592267	2.001290	0.001289	-3.03158
0.40	1.591878	2.000843	0.000842	-2.93651
0.50	1.591337	2.000425	0.000425	-2.80853
0.60	1.590694	2.000054	0.000054	-2.64144
0.70	1.590002	1.999918	-0.000082	-2.42443
0.80	1.589334	1.999713	-0.000287	-2.13627
0.90	1.588691	1.999857	-0.000143	-1.72233

TABLE II. SERIES $\theta = 0$

e_0	a_0/a'	T/T'	σ	$k/n'^2a'^2$
0.0125	1.567251	2.049331	0.047012	-3.14039
0.025	1.578107	2.027381	0.026651	-3.14414
0.05	1.582210	2.018634	0.018293	-3.14372
0.10	1.582021	2.016625	0.016354	-3.13595
0.20	1.571565	2.027216	0.026495	-3.09613
0.30	1.572116	2.100643	0.091440	-3.04340
0.40	1.781873	2.122834	0.109396	-3.03192

TABLE III. ASYMMETRIC SERIES

e_0	a_0/a'	T/T'	σ	$k/n'^2a'^2$	$\theta_0/2\pi$
0.0375	1.595500	1.995546	-0.004474	-3.14923	0.4250
0.05	1.595583	1.995517	-0.004503	-3.14787	0.3578
0.10	1.595884	1.995702	-0.004317	-3.13843	0.2790
0.15	1.596251	1.995968	-0.004051	-3.12259	0.2461
0.20	1.596523	1.99634	-0.00368	-3.10018	0.2243
0.25	1.596600	1.99681	-0.00320	-3.07101	0.2074
0.30	1.596398	1.99732	-0.00268	-3.03479	0.1953
0.40	1.595265	1.99851	-0.00149	-2.93994	0.1813

TABLE IV. ORBITS WITH $\theta = \pi$; REPRESENTATION OF x AND y BY $x = \Sigma a_j \cos ju$; $y = \Sigma b_j \sin ju$

j	a_j	b_j	a_j	b_j	a_j	b_j
	$e_0 = 0.0125$		$e_0 = 0.025$		$e_0 = 0.0375$	
0	-0.01126		-0.02130		-0.031361	
1	+1.59773	-1.59771	+1.59149	-1.59085	+1.589103	-1.587454
2	+0.03455	-0.03456	+0.06455	-0.06457	+0.094620	-0.094680
3	+0.00103	-0.00102	+0.00111	-0.00111	+0.001238	-0.001234
4	+0.00021	-0.00021	+0.00021	-0.00021	+0.000202	-0.000201
5	+0.00007	-0.00007	+0.00007	-0.00007	+0.000067	-0.000066
6	+0.00003	-0.00003	+0.00003	-0.00003	+0.000026	-0.000026
7	+0.00001	-0.00001	+0.00001	-0.00001	+0.000011	-0.000011
8					+0.000005	-0.000005
9					+0.000003	-0.000003
10					+0.000001	-0.000001
	$e_0 = 0.05$		$e_0 = 0.10$		$e_0 = 0.15$	
0	-0.04141		-0.08108		-0.119671	
1	+1.58770	-1.58466	+1.58468	-1.57234	+1.581610	-1.554355
2	+0.12469	-0.12483	+0.24442	-0.24553	+0.363032	-0.366707
3	+0.00142	-0.00141	+0.00286	-0.00274	+0.005519	-0.004971
4	+0.00019	-0.00019	+0.00009	-0.00010	-0.000144	+0.000059
5	+0.00006	-0.00006	+0.00006	-0.00005	+0.000070	-0.000055
6	+0.00002	-0.00002	+0.00002	-0.00002	+0.000013	-0.000015
7	+0.00001	-0.00001	+0.00001	-0.00001	+0.000008	-0.000007
8					+0.000003	-0.000003
9					+0.000002	-0.000001
10					+0.000001	-0.000001
	$e_0 = 0.20$		$e_0 = 0.25$		$e_0 = 0.30$	
0	-0.156754		-0.191935		-0.22482	
1	+1.577374	-1.530065	+1.571465	-1.499623	+1.56344	-1.46337
2	+0.480251	-0.488704	+0.595881	-0.611805	+0.70980	-0.73618
3	+0.009597	-0.007933	+0.015327	-0.011445	+0.02296	-0.01532
4	-0.000619	+0.000273	-0.001498	+0.000496	-0.00299	+0.00064
5	+0.000137	-0.000061	+0.000320	-0.000050	+0.00072	+0.00003
6	-0.000004	-0.000013	-0.000054	+0.000021	-0.00018	-0.00007
7	+0.000009	-0.000005	+0.000021	+0.000000	+0.00006	+0.00002
8	+0.000002	-0.000003	-0.000002	-0.000004	-0.00002	-0.00001
9	+0.000001	-0.000001	+0.000002		+0.000001	
10	+0.000001	-0.000001				

TABLE IV (Continued)

a_j	b_j	a_j	b_j
$e_0 = 0.40$		$e_0 = 0.50$	
-0.28224		-0.32616	
+1.53895	-1.37501	+1.49931	-1.26871
+0.93261	-0.98902	+1.14980	-1.24640
+0.04504	-0.02371	+0.07748	-0.03301
-0.00877	+0.00021	-0.01991	-0.00182
+0.00282	+0.00076	+0.00804	+0.00304
-0.00105	-0.00050	-0.00375	-0.00206
+0.00043	+0.00025	+0.00190	+0.00123
-0.00018	-0.00012	-0.00100	-0.00072
+0.00008	+0.00006	+0.00054	+0.00041
-0.00004	-0.00003	-0.00030	-0.00024
+0.00002	+0.00001	+0.00017	+0.00014
-0.00001	-0.00001	-0.00010	-0.00008
		+0.00006	+0.00004
		-0.00004	-0.00002
		+0.00004	
$e_0 = 0.60$		$e_0 = 0.70$	
-0.3539		-0.3630	
+1.4388	-1.1481	+1.3505	-1.0161
+1.3643	-1.5058	+1.5804	-1.7631
+0.1210	-0.0453	+0.1755	-0.0659
-0.0378	-0.0052	-0.0624	-0.0076
+0.0180	+0.0075	+0.0334	+0.0131
-0.0099	-0.0057	-0.0206	-0.0113
+0.0058	+0.0039	+0.0136	+0.0087
-0.0036	-0.0026	-0.0093	-0.0066
+0.0022	+0.0017	+0.0066	+0.0050
-0.0014	-0.0012	-0.0048	-0.0037
+0.0009	+0.0008	+0.0035	+0.0028
-0.0006	-0.0005	-0.0026	-0.0021
+0.0004	+0.0004	+0.0020	+0.0016
-0.0003	-0.0002	-0.0015	-0.0012
+0.0002	+0.0002	+0.0012	+0.0009
-0.0001	-0.0001	-0.0009	-0.0007
+0.0001	+0.0001	+0.0007	+0.0005
-0.0001	-0.0001	-0.0006	-0.0003
+0.0001	+0.0001	+0.0005	+0.0002
		-0.0005	-0.0001
		+0.0005	

series with argument

$$u = \frac{2\pi}{T} (t - t_0),$$

(4)

where t_0 is the time of conjunction, so that u is the uniform part of $\lambda' - \lambda$. A program was devised to carry out Fourier analysis of these quantities, which, reading the values at successive steps of integration from the output cards from the integration of the equations of motion, determined the values of x , y , dx/dt and dy/dt at N equally spaced values of u in the range $0 \leq u < 2\pi$, by interpolation using Neville's process. The Fourier analysis was then carried out on those values. N was given the value 30 for most of the orbits analyzed and 20 for those with the smaller values of e_0 . The Fourier series found for the coordinates are given in Tables IV to VI.

The stability of the orbits. In order to investigate the ordinary linear stability of a selection of the orbits obtained by numerical integration, an appropriate form of Hill's equation was derived, as follows. The equations of motion in the plane $z = 0$ may be written, from Eq (1) in the form

$$\frac{d^2x}{dt^2} - 2n' \frac{dy}{dt} = \frac{\partial \Omega}{\partial x}$$
$$\frac{d^2y}{dt^2} + 2n' \frac{dx}{dt} = \frac{\partial \Omega}{\partial y},$$

(5)

where

$$\Omega = \frac{\mu}{r_s} + \frac{\mu m'}{r_J} + n'^2 (x^2 + y^2)$$

and Jacobi's integral is then

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 2\Omega + C,$$

(6)

where C is constant. If $x = x_0(t)$, $y = y_0(t)$ is a known periodic solution of these equations, consider a solution $x = x_0 + \Delta x$, $y = y_0 + \Delta y$, where Δx and Δy are supposed initially small. Then, neglecting squares and products of these small quantities, the Eqs. (5) give

$$\frac{d^2}{dt^2} (\Delta x) - 2n' \frac{d}{dt} (\Delta y)$$
$$= \left(\frac{\partial^2 \Omega}{\partial x^2}\right)_0 \Delta x + \left(\frac{\partial^2 \Omega}{\partial y \partial x}\right)_0 \Delta y$$

(7)

$$\frac{d^2}{dt^2} (\Delta y) + 2n' \frac{d}{dt} (\Delta x)$$
$$= \left(\frac{\partial^2 \Omega}{\partial x \partial y}\right)_0 \Delta x + \left(\frac{\partial^2 \Omega}{\partial y^2}\right)_0 \Delta y,$$

where the suffix "0" applied to a function means that it is evaluated for $x = x_0$, $y = y_0$, and $z = 0$. From Jacobi's integral, taking $\Delta C = 0$ (which does not lose generality since the solution x_0 , y_0 may be chosen appropriately from the series),

$$\frac{dx_0}{dt} \cdot \frac{d}{dt} (\Delta x) + \frac{dy_0}{dt} \cdot \frac{d}{dt} (\Delta y)$$
$$= \left(\frac{d^2 x_0}{dt^2} - 2n' \frac{dy_0}{dt}\right) \Delta x$$
$$+ \left(\frac{d^2 y_0}{dt^2} + 2n' \frac{dx_0}{dt}\right) \Delta y.$$

(8)

TABLE V. ORBITS WITH $\theta = 0$; REPRESENTATION OF x AND y BY $x = \sum a_j \cos ju$; $y = \sum b_j \sin ju$

j	a_j	b_j	a_j	b_j	a_j	b_j
	$e_0 = 0.0125$		$e_0 = 0.025$		$e_0 = 0.05$	
0	0.00958		0.01963		0.04001	
1	+1.56246	-1.56268	+1.57323	-1.57289	+1.57732	-1.57472
2	-0.02731	+0.02727	-0.05722	+0.05719	-0.11787	+0.11793
3	+0.00149	-0.00150	+0.00157	-0.00158	+0.00213	-0.00214
4	+0.00031	-0.00031	+0.00031	-0.00031	+0.00035	-0.00035
5	+0.00010	-0.00010	+0.00010	-0.00010	+0.00012	-0.00011
6	+0.00004	-0.00004	+0.00004	-0.00004	+0.00005	-0.00005
7	+0.00002	-0.00002	+0.00002	-0.00002	+0.00002	-0.00002
8	+0.00001	-0.00001	+0.00001	-0.00001	+0.00001	-0.00001
9			+0.00001		+0.00001	
	$e_0 = 0.10$		$e_0 = 0.20$		$e_0 = 0.30$	
0	0.08091		0.16365		0.25596	
1	+1.57678	-1.56499	+1.56404	-1.51419	+1.51352	-1.39364
2	-0.23995	+0.24090	-0.48925	+0.49801	-0.73831	+0.77010
3	+0.00433	-0.00425	+0.01521	-0.01357	+0.05576	-0.04665
4	+0.00052	-0.00052	+0.00185	-0.00153	+0.00870	-0.00590
5	+0.00016	-0.00016	+0.00048	-0.00041	+0.00246	-0.00152
6	+0.00006	-0.00006	+0.00017	-0.00016	+0.00086	-0.00051
7	+0.00003	-0.00003	+0.00007	-0.00007	+0.00034	-0.00019
8	+0.00001	-0.00001	+0.00003	-0.00003	+0.00014	-0.00007
9	+0.00001	-0.00001	+0.00002	-0.00002	+0.00006	-0.00002
10			+0.00001	-0.00001	+0.00003	-0.00001
11					+0.00001	
12					+0.00001	

TABLE VI. ASYMMETRIC SERIES

$$x = \sum a_j \cos ju + \sum b_j \sin ju \quad y = \sum a_j' \cos ju + \sum b_j' \sin ju$$

j	a_j	b_j	a_j'	b_j'	a_j	b_j	a_j'	b_j'
	$e_0 = 0.0375$				$e_0 = 0.05$			
0	-0.02810		+0.01326		-0.02654		0.03034	
1	+1.58923	+0.00156	+0.00016	-1.58828	+1.58761	+0.00341	+0.00039	-1.58822
2	+0.08485	+0.04007	+0.04012	-0.08486	+0.08010	+0.09176	+0.09182	-0.07998
3	+0.00113	+0.00030	+0.00030	-0.00113	+0.00085	+0.00066	+0.00066	-0.00085
4	+0.00021	+0.00002	+0.00002	-0.00020	+0.00021	+0.00005	+0.00005	-0.00021
5	+0.00007	+0.00001	+0.00001	-0.00007	+0.00006	+0.00002	+0.00002	-0.00006
6	+0.00003			-0.00003	+0.00002	+0.00001	+0.00001	-0.00002
7	+0.00001			-0.00001	+0.00001			-0.00001
8	+0.00001			-0.00001				
	$e_0 = 0.10$				$e_0 = 0.15$			
0	-0.01662		0.07636		-0.00027		0.11571	
1	+1.57665	+0.00686	+0.00216	-1.58741	+1.55931	+0.00625	+0.00635	-1.58507
2	+0.04956	+0.23244	+0.23158	-0.04896	-0.00190	+0.35601	+0.35261	+0.00185
3	-0.00094	+0.00113	+0.00120	+0.00101	-0.00349	+0.00047	+0.00046	+0.00398
4	+0.00020	+0.00017	+0.00018	-0.00020	+0.00010	+0.00034	+0.00042	-0.00010
5	+0.00005	+0.00004	+0.00004	-0.00005	+0.00004	+0.00006	+0.00006	-0.00005
6	+0.00002	+0.00002	+0.00002	-0.00002		+0.00002	+0.00002	
7	+0.00001	+0.00001	+0.00001	-0.00001		+0.00001	+0.00001	
	$e_0 = 0.20$				$e_0 = 0.25$			
0	0.02081		0.15086		0.04548		0.18177	
1	+1.53675	+0.00113	+0.01358	-1.57989	+1.51018	-0.00861	+0.02409	-1.57053
2	-0.07045	+0.47081	+0.46366	+0.06720	-0.15412	+0.57720	+0.56627	+0.14397
3	-0.00642	-0.00126	-0.00208	+0.00769	-0.00960	-0.00396	-0.00696	+0.01152
4	-0.00008	+0.00053	+0.00076	+0.00028	-0.00029	+0.00077	+0.00105	+0.00115
5	+0.00003	+0.00007	+0.00012	-0.00007	+0.00004	+0.00005	+0.00029	-0.00006
6	-0.00001	+0.00002	+0.00001		-0.00002		-0.00001	-0.00004
7	-0.00001	+0.00001	+0.00001		-0.00001			
	$e_0 = 0.30$							
0	0.0702		0.2091					
1	+1.4800	-0.0218	+0.0368	-1.5565				
2	-0.2430	+0.6778	+0.6639	+0.2222				
3	-0.0132	-0.0072	-0.0141	+0.0150				
4	-0.0005	+0.0012	+0.0010	+0.0026				
5	+0.0002	+0.0000	+0.0006	+0.0001				
6		-0.0001	+0.0001	-0.0001				

ut

$$\frac{dx_0}{dt} = V \cos \psi, \quad \frac{dy_0}{dt} = V \sin \psi, \quad (9)$$

nd

$$\begin{aligned} p &= \Delta x \cos \psi + \Delta y \sin \psi \\ q &= -\Delta x \sin \psi + \Delta y \cos \psi. \end{aligned} \quad (10)$$

hen p and q are respectively the tangential and normal displacements from the periodic orbit. Eliminating Δx and Δy in favor of p and q in eqs. (7), using (8), gives

$$\frac{d}{dt} \left(\frac{p}{V} \right) = 2 \left(n' + \frac{d\psi}{dt} \right) \frac{q}{V} \quad (11)$$

nd

$$\frac{d^2 q}{dt^2} + f(t) \cdot q = 0, \quad (12)$$

here

$$\begin{aligned} f(t) &= 4n'^2 + 6n' \frac{d\psi}{dt} + 3 \left(\frac{d\psi}{dt} \right)^2 - \left(\frac{\partial^2 \Omega}{\partial x^2} \right)_0 \sin^2 \psi \\ &\quad + 2 \left(\frac{\partial^2 \Omega}{\partial x \partial y} \right)_0 \sin \psi \cos \psi - \left(\frac{\partial^2 \Omega}{\partial y^2} \right)_0 \cos^2 \psi. \end{aligned}$$

q. (12) is Hill's equation. Now in the case of those orbits for which we have the coordinates as Fourier series, their higher derivatives with respect to the time are more readily obtained than are the partial derivatives of Ω . Differentiating Eq. (5) with respect to the time we see that

$$\frac{dx_0}{dt^3} - 2n' \frac{d^2 y_0}{dt^2} = \left(\frac{\partial^2 \Omega}{\partial x^2} \right)_0 \frac{dx_0}{dt} + \left(\frac{\partial^2 \Omega}{\partial y \partial x} \right)_0 \frac{dy_0}{dt} \quad (13)$$

$$\frac{dy_0}{dt^3} + 2n' \frac{d^2 x_0}{dt^2} = \left(\frac{\partial^2 \Omega}{\partial x \partial y} \right)_0 \frac{dx_0}{dt} + \left(\frac{\partial^2 \Omega}{\partial y^2} \right)_0 \frac{dy_0}{dt},$$

nd since

$$\frac{\partial^2 \Omega}{\partial x^2} + \frac{\partial^2 \Omega}{\partial y^2} + \frac{\partial^2 \Omega}{\partial z^2} = 2n'^2,$$

e have

$$f(t) = 2n'^2 + 4n' \frac{d\psi}{dt} + 3 \left(\frac{d\psi}{dt} \right)^2 + X + \left(\frac{\partial^2 \Omega}{\partial z^2} \right)_0,$$

here

$$V^2 X = \frac{dx_0}{dt} \frac{d^3 x_0}{dt^3} + \frac{dy_0}{dt} \frac{d^3 y_0}{dt^3}.$$

Now

$$\begin{aligned} \left(\frac{\partial^2 \Omega}{\partial z^2} \right)_0 &= - \left(\frac{\mu}{r_s^3} \right)_0 - \left(\frac{\mu m'}{r_j^3} \right)_0 \\ &= \frac{1}{y_0} \left(\frac{d^2 y_0}{dt^2} + 2n' \frac{dx_0}{dt} - n'^2 y_0 \right), \end{aligned}$$

and so, putting

$$y_0 Y = \frac{d^2 y_0}{dt^2} + 2n' \frac{dx_0}{dt},$$

we have

$$\begin{aligned} f(t) &= n'^2 + 4n' \frac{d\psi}{dt} + 3 \left(\frac{d\psi}{dt} \right)^2 + X + Y \\ &= (n - n')^2 \Theta(u), \text{ say.} \end{aligned} \quad (14)$$

Then Hill's equation (12) becomes

$$\frac{d^2 q}{du^2} + \Theta(u) \cdot q = 0. \quad (12')$$

Darwin (1909) showed that any solution of (12') was necessarily of the form

$$q = A \cdot \exp(icu) \cdot Q_1(u) + B \cdot \exp(-icu) \cdot Q_2(u), \quad (15)$$

where $Q_1(u)$ and $Q_2(u)$ are periodic functions of period 2π , A and B are disposable constants, and c is a constant which depends on the function $\Theta(u)$. If c is real, q , and therefore, from (11), also p , contain only sine and cosine terms in u and therefore $|p|$ and $|q|$ remain small if initially so, and the periodic orbit is stable in the ordinary sense, while if c is complex, real exponential functions of u appear in p and q as factors, so that in this case the periodic orbit is unstable.

Treating the orbit as of fixed eccentricity e , I find to the first order in e and σ

$$\Theta(u) = 1 + 2\sigma + 12e \cos \theta \cdot \cos u.$$

For most of the orbits $|\sigma|$ is considerably smaller than e , and so the constant term in Θ differs from unity by a quantity less than the coefficient of $\cos u$, in general. For this reason the method given by Brown (1936) for the determination of c cannot be used without modification since some of the quantities involved in the successive approximations contain the very small divisor σ .

Since Θ in our orbits is not a cosine series in even multiples of u Baker's (1915) solution is not applicable. Θ is a function of x_0 and y_0 and their derivatives, and these are functions of u with period 2π , so we may write

$$\begin{aligned}\Theta(u) &= 1 + \theta_0 + 2 \sum_{r=1}^{\infty} \{a_r \cos ru + b_r \sin ru\} \\ &= 1 + \theta_0 + \sum_{r=1}^{\infty} \{ (a_r - ib_r) \exp(iru) \\ &\quad + (a_r + ib_r) \exp(-iru) \} \\ &= 1 + \sum_{r=-\infty}^{\infty} \theta_r \exp(iru), \text{ say.} \quad (16)\end{aligned}$$

The quantities $|\theta_r|$, except for θ_0 , are found in general to diminish with increasing $|r|$.

Consider the case of the solution (15) in which $B = 0$. It may be written

$$\begin{aligned}q_0 &= -v_0[q_1\{\theta_{-1} - v_2\theta_{1-2} + (v_0v_{-2} + v_0v_2 + v_2v_3)\theta_{-1}\theta_{2-2} - (v_{-2} + v_3)\theta_{2-3} - v_2^2v_3\theta_{1-1}\theta_{-2} \\ &\quad + v_2v_3\theta_{1-2}\theta_{-3} + \dots\} + q_{-1}\{\theta_1 - v_{-2}\theta_{-1}\theta_2 + (v_0v_{-2} + v_0v_2 + v_{-2}v_{-3})\theta_1\theta_{-2}\theta_2 \\ &\quad - (v_2 + v_{-3})\theta_{-2}\theta_3 - v_{-2}^2v_{-3}\theta_{1-1}\theta_2 + v_{-2}v_{-3}\theta_{-1}\theta_3 + \dots\}] \\ q_2 &= -v_2[q_1\{\theta_1 - (v_0 + v_3)\theta_{-1}\theta_2 + v_2v_3\theta_{-1}\theta_1^2 + (v_0v_2 + v_2v_4 + v_3v_4)\theta_1\theta_2\theta_{-2} - v_4\theta_{-2}\theta_3 \\ &\quad - v_2v_3v_4\theta_{1-1}\theta_{-2} + \dots\} + q_{-1}\{-v_0\theta_1\theta_2 + v_0v_{-2}\theta_{-1}\theta_2^2 + \theta_3 - v_{-2}\theta_{-1}\theta_4 + \dots\}] \\ q_{-2} &= -v_{-2}[q_1\{-v_0\theta_{-1}\theta_{-2} + v_2v_0\theta_{1-2}\theta_{-2}^2 + \theta_{-3} - v_2\theta_{1-4} + \dots\} + q_{-1}\{\theta_{-1} - (v_0 + v_{-3})\theta_{1-2} \\ &\quad + v_{-2}v_{-3}\theta_{1-1}\theta_{-1}^2 + (v_0v_{-2} + v_{-2}v_{-4} + v_{-3}v_{-4})\theta_{-1}\theta_{2-2} - v_{-4}\theta_{2-3} - v_{-2}v_{-3}v_{-4}\theta_{-1}\theta_{-2} + \dots\}] \quad (18) \\ q_3 &= -v_3[q_1\{\theta_2 - v_2\theta_1^2 - (v_0 + v_4)\theta_{-1}\theta_3 + (v_0v_2 + v_2v_3 + v_2v_4 + v_3v_4)\theta_1\theta_{1-2} \\ &\quad - (v_2 + v_4)v_2v_3\theta_{1-1}\theta_{-1} + \dots\} + q_{-1}\{-v_0\theta_{1-3} + v_2v_0\theta_{1-2}\theta_2 - v_2\theta_{1-3}\theta_3 + \theta_4 + \dots\}] \\ q_{-3} &= -v_{-3}[q_1\{-v_0\theta_{-1}\theta_{-3} + v_0v_{-2}\theta_{1-2}\theta_{-2} - v_{-2}\theta_{-1}\theta_{-3} + \theta_{-4} + \dots\} + q_{-1}\{\theta_{-2} - (v_0 + v_{-4})\theta_{1-3} \\ &\quad - v_{-2}\theta_{-1}^2 + (v_0v_{-2} + v_{-2}v_{-3} + v_{-2}v_{-4} + v_{-3}v_{-4})\theta_{1-1}\theta_{-2} - v_{-2}v_{-3}(v_{-2} + v_{-4})\theta_{1-1}\theta_{-1}^3 + \dots\}] \\ q_4 &= -v_4q_1\{\theta_3 - (v_2 + v_3)\theta_{1-2} + v_2v_3\theta_{1-1}^3\} \\ q_{-4} &= -v_{-4}q_{-1}\{\theta_{-3} - (v_{-2} + v_{-3})\theta_{-1}\theta_{-2} + v_{-2}v_{-3}\theta_{-1}^3\}.\end{aligned}$$

The equations (17) with $r = 1$ and $r = -1$ are then, to sixth order,

$$\begin{aligned}\{1 + \lambda_{-1} - (c - 1)^2\}q_{-1} + v_{-1}q_1 &= 0, \\ v_1q_{-1} + \{1 + \lambda_1 - (c + 1)^2\}q_1 &= 0, \quad (19)\end{aligned}$$

where

$$\begin{aligned}\lambda_{-1} &= \theta_0 - (v_0 + v_{-2})\theta_1\theta_{-1} + v_{-2}(v_0 + v_{-3})\theta_{-1}\theta_2 + v_{-2}(v_0 + v_{-3})\theta_1\theta_{-2} - v_{-2}^2v_{-3}\theta_{1-1}\theta_{-1}^2 - v_{-3}\theta_{2-2} \\ &\quad - (v_0^2v_{-2} + v_0^2v_2 + 2v_0v_{-2}v_{-3} + v_0v_{-2}^2 + v_{-2}^2v_{-4} + 2v_{-2}v_{-3}v_{-4} + v_{-2}v_{-3}^2 + v_{-3}^2v_{-4})\theta_1\theta_{-1}\theta_{2-2} \\ &\quad + v_{-2}v_{-3}(v_0v_{-2} + v_{-2}v_{-4} + v_{-2}v_{-3} + v_{-3}v_{-4})\theta_{1-1}\theta_{-1}^3\theta_2 + (v_0v_2 + v_0v_{-3} + v_{-2}v_{-4} + v_{-3}v_{-4})\theta_{-1}\theta_{-2}\theta_3 \\ &\quad - v_{-2}v_{-3}(v_0 + v_{-4})\theta_{-1}\theta_3 + (v_{-2}v_{-4} + v_0v_{-3} + v_{-3}v_{-4} + v_0v_2)\theta_1\theta_{2-3} - (v_2 + v_{-4})\theta_3\theta_{-3}, \\ v_{-1} &= \theta_{-2} - v_0\theta_{-1}^2 + v_0(v_2 + v_{-2})\theta_1\theta_{-1}\theta_{-2} - (v_{-2} + v_2)\theta_{1-3} - v_0v_2v_{-2}\theta_{1-2}\theta_{-2}^2 - v_0(v_0v_2 + v_0v_{-2} \\ &\quad + v_2v_3 + v_{-2}v_{-3})\theta_{-1}\theta_{2-2} + (v_0v_2 + v_0v_{-2} + v_0v_3 + v_0v_{-3} + v_2v_3 + v_{-2}v_{-3})\theta_{-1}\theta_{2-3} \\ &\quad + v_0v_2^2v_3\theta_{1-1}\theta_{-1}^2\theta_{-2} - v_2v_3(v_0 + v_2)\theta_{-1}\theta_{-1}\theta_{-3} + v_2(v_{-2} + v_3)\theta_{1-1}\theta_{-4} - (v_3 + v_{-3})\theta_{2-4}.\end{aligned}$$

$$q = \sum_{r=-\infty}^{\infty} q_r \exp\{i(r+c)u\},$$

so that, on equating coefficients in (12'),

$$u_r q_r = - \sum_{s=1}^{\infty} (\theta_s q_{r-s} + \theta_{-s} q_{r+s}) \quad (r = 0, \pm 1, \pm 2, \dots) \quad (17)$$

where $u_r = 1 + \theta_0 - (r+c)^2$. Brown solved these equations by successive approximation. But in our case θ_0 is small, so that since c also proves to be small u_1 and u_{-1} are small and we lose accuracy if we divide by them. Regarding $|\theta_r|$ as of the r th order of small quantities, and solving the equations (17), except those with $r = 1$ and $r = -1$, by successive approximation up to the sixth order in q_0 , q_2 and q_{-2} , to the fourth in q_3 and q_{-3} , and to the third in q_4 and q_{-4} I find, writing $v_r = 1/u_r$,

The expression for λ_1 is obtained from that for λ_{-1} by replacing v_r by v_{-r} , and θ_r by θ_{-r} , and the expression for ν_1 is obtained from that for ν_{-1} in the same way. If q_1 and q_{-1} are not both zero, that is to say if the solution is not to be identically zero, we must have

$$\left| \begin{array}{cc} 1 + \lambda_{-1} - (c - 1)^2 & \nu_{-1} \\ \nu_1 & 1 + \lambda_1 - (c + 1)^2 \end{array} \right| = 0,$$

that is to say

$$-(4 + \lambda_{-1} + \lambda_1)c^2 + 2(\lambda_1 - \lambda_{-1})c + \lambda_1\lambda_{-1} - \nu_1\nu_{-1} = 0. \quad (20)$$

From this equation c may be determined by successive approximation, beginning by calculating u_r with $c = 0$. This was done for a selection of the orbits with smaller eccentricities, and the functions Θ and the values of c determined from them are given in Table VII.

TABLE VII. THE FUNCTION $\Theta(u)$ AND VALUES OF c

Symmetric orbits: $\Theta(u) = \Sigma A_j \cos ju$				
e_0	$e_0 = 0.0125$	0.05	0.10	
0	1.101	1.065	1.163	
1	+0.140	+0.606	+1.296	
2	-0.029	+0.079	+0.463	
3	-0.021	-0.011	+0.125	
4	-0.014	-0.018	+0.017	
5	-0.009	-0.013	-0.011	
6	-0.006	-0.009	-0.015	
7	-0.004	-0.006	-0.012	
8	-0.003	-0.005	-0.009	
9		-0.001	-0.003	
c	0.0470	0.025 <i>i</i>	0.070 <i>i</i>	
Asymmetric orbits: $\Theta = \Sigma A_j \cos ju + \Sigma B_j \sin ju$				
e_0	$e_0 = 0.0125$	0.0375	0.05	0.10
0	0.969	1.011	1.039	1.143
1	-0.171	-0.479	-0.638	-1.331
2	-0.012	+0.055	+0.108	+0.546
3	-0.009	-0.018	-0.032	-0.205
4	-0.006	-0.004	+0.005	+0.070
5	-0.005	-0.004		-0.026
c	0.0171	0.000	0.018 <i>i</i>	0.019 <i>i</i>
$\theta_0/2\pi = 0.3578$ $e_0 = 0.05$				
j	A_j	B_j	A_j	B_j
0	1.022		1.077	
1	-0.409	-0.469	-0.273	-1.171
2	-0.038	+0.121	-0.444	+0.197
3	+0.011	-0.011	-0.079	+0.164
4	-0.010	-0.002	+0.087	-0.016
5	-0.003	-0.000	+0.014	-0.070
6	-0.002	-0.001	-0.051	-0.013
7	-0.001		+0.015	+0.006
8	-0.001		+0.010	
c	0.015		0.062	

Both the symmetric orbits with $e_0 = 0.0125$ are stable, the symmetric orbits of both series with $e_0 = 0.05$ and with $e_0 = 0.10$ are unstable, and the asymmetric orbits with $e_0 = 0.05$ and $e_0 = 0.10$ are stable. The symmetric orbit with $\theta_0 = 0$ and $e_0 = 0.0375$ has a value of c too close to zero to be determined from the data available.

For orbits of larger eccentricity the successive terms in the Fourier series for Θ do not diminish rapidly and this method becomes lengthy. Darwin (1909) gave a method for the determination of c using the numerical integration of two particular solutions of Hill's equation. If $q = \phi(u)$ and $g = \psi(u)$ are two solutions such that

$$\phi(0) = 1, \quad \phi'(0) = 0,$$

$$\psi(0) = 0, \quad \psi'(0) = 1,$$

then by a simple extension of Darwin's treatment to include cases where $\Theta(u)$ is not an even function I find that

$$\cos 2\pi c = \frac{1}{2} \{ \phi(\pi)\psi'(-\pi) + \phi(-\pi)\psi'(\pi) - \phi'(\pi)\psi(-\pi) - \phi'(-\pi)\psi(\pi) \}. \quad (21)$$

This method was applied to the asymmetric orbit with $e_0 = 0.05$. Θ was determined for values of u at 10° intervals in the range $0^\circ \leq u < 360^\circ$ and the solutions ϕ and ψ were found numerically. The quantity $\phi\psi' - \phi'\psi$ is independent of u and provides a check on the calculations. The values of ϕ , ψ , ϕ' and ψ' found for $u = -180^\circ$, 0° and 180° are given in Table VIII. Hence $c = 0.011$, in fair agreement with the previous result. For the orbits with $e_0 = 0.10$, however, it was found easier to obtain results of any accuracy by the previous method than by this one.

TABLE VIII. CHECK ON THE CALCULATION OF THE ASYMMETRIC ORBITS WITH $e_0 = 0.05$

u	ϕ	ϕ'	ψ	ψ'	$\phi\psi' - \psi\phi'$
$-\pi$	-0.7566	-0.2239	0.5659	-1.1548	+1.0004
0	1.0000	0.0000	0.0000	1.0000	+1.0000
π	-0.6189	-0.3598	0.6764	-1.2230	+1.0002

The ordinary stability of these orbits may also be investigated by the use of the secular and critical terms of the disturbing function. Such an investigation, considering orbits of eccentricities up to 0.10, leads to the conclusion that those orbits of both symmetrical series are stable which have small eccentricities, those on the series $\theta = 0$ being unstable which have a mean eccentricity greater than 0.0316, those on the series $\theta = \pi$ being unstable which have a mean value of e greater than 0.0367, the value at the point

of bifurcation with the asymmetric series, and the orbits of the latter series are stable. Thus there is an exchange of stability at the point of bifurcation. The present work is in good agreement with these results.

The investigation was carried out at the Yale University Observatory, and was supported financially with funds provided from a contract with the Office of Naval Research.

It is a pleasure to record my indebtedness to Professor Brouwer for suggesting this investigation, for putting the facilities at my disposal which enabled me to carry it out and for much valuable guidance in the course of it, and also to Dr. G. A. Wilkins and Dr. M. S. Davis for many helpful and stimulating discussions.

REFERENCES

- Baker, H. F. 1915, *Phil. Trans. A* 216 (Part 1), 129.
 Brown, E. W. 1936, *A. J.* 45, 84.
 Darwin, G. H. 1898, *Acta Math.* 21, 99.
 ——. 1909, *M. N.* 70, 108; *Scientific Papers IV*, 143.
 ——. 1912, *M. N.* 72, 642.
 Hill, G. W. 1902, *A. J.* 22, 93, 117.
 Kopal, Z. 1955, *Numerical Analysis* (New York: John Wiley), p. 38.
 Message, P. J. 1958, *A. J.* 63, 443.
 Neville, E. H. 1934, *J. Indian Math. Soc.* 20, 87; and see especially H. M. Nautical Almanac Office 1956, *Interpolation and Allied Tables* (London: H. M. Stationery office), p. 59.
 Poincaré, H. 1892, *Méthodes Nouvelles de la Mécanique Céleste* (Paris: Gauthier-Villars) I, chap. 3.
 Schwarzschild, K. 1903, *A. N.* 160, 385.
 van Veen, S. C. 1927, *Periodieke Oplossingen in Communicaatgebieden* (Amsterdam: H. J. Paris).

PARALLAX, ORBITAL MOTION AND MASS OF THE VISUAL BINARY L726-8

By PETER VAN DE KAMP

Sproul Observatory, Swarthmore College, Swarthmore, Pennsylvania

Received March 2, 1959

Abstract. Measures on photographs taken with the 24-inch Sproul refractor over the interval 1949 to 1959 yielded $+".370 \pm ".010$ (p.e.) for the relative parallax. The double star measurements are well represented by an orbit with period 200 years and semi-axis major $5".57$, appreciably larger dimensions than estimated earlier. The sum of the masses is $0.079\odot$, confirming Luyten's deduction that these are the visual binary components of smallest known mass; the individual masses are taken as $.044\odot$ and $.035\odot$.

Introduction. L726-8, $1^h 34^m 0^s$, $-18^\circ 28'$ (1900) is a visual binary with photovisual magnitudes 12.45 and 12.95 (Luyten 1949) and spectrum dM5.5e for each of the components (Joy and Humason 1949). This star of large proper motion ($3".31$ in 80.5) was discovered by Luyten and its duplicity noted shortly thereafter. The rapid orbital motion observed during the first few years after the star's discovery led to estimates of orbits with semi-axes major and periods ranging from $1".7$, 32 years (Luyten 1950) to $2".3$, 57 years (Luyten 1954). Provisional determinations of the parallax by Steward, Yerkes, and Sproul Observatories yielded respectively the values $+".56 \pm ".07$, $+".41 \pm ".04$, and $+".37 \pm ".04$ (van de Kamp and Lippincott 1950). Wagman (1956) has derived a provisional value of $+".333 \pm ".019$. Assuming extreme values of $0".4$ and $0".3$ for the parallax, lower and upper limits of $0.06\odot$ and $0.20\odot$ were found for the combined mass (Luyten 1954). A new Yerkes value for the parallax $+".381 \pm ".010$ appeared to narrow down the permissible range for the total mass, for

which, using Protitch's (1955) orbit, $0.079\odot$ was found (Luyten 1956). Thus the components of L726-8 appeared to have only half the mass of Ross 614B, whose mass, as an unseen companion was first estimated at $0.08\odot$ (Lippincott 1951; van de Kamp 1954b), which value was confirmed later when the star was first seen (Lippincott 1955a, 1955b). For Ross 614B the parallax was known with high accuracy ($+".251 \pm ".003$) and could hardly affect the calculated mass, the only uncertainty arising from error in the measured separation ($1".19$) with the 200-inch reflector. An upward correction to $1".4$, which is perhaps the extreme permissible, would result in a value of $.10\odot$ for the mass of Ross 614B. The statement in 1955 that Ross 614B was the star of smallest known mass was based on the comparatively adequate accuracy of the contributing parameters: parallax, period, and semi-axis major, which for L726-8 had a fairly wide range of permissible error at that time.

Parallax. The procedure described in previous Sproul papers has been followed (van de Kamp

TABLE I. OBSERVING DATA, MEASURED POSITIONS AND RESIDUALS

epoch	Date	H.A. min.	No. of pl. exp.	Obs.	P_α	P_δ	X unit .0001 mm	Y unit .0001 mm	v_x unit .0001 mm	v_y unit .0001 mm	p
8.9026	Nov. 25	- 2	1 2	Bi	-.563	-.561	-9 6214	-2 0798	+105	-24	.2
9.6015	Aug. 8	-32	2 4	Dt	+.891	+.144	4723	0516	+101	-83	.5
.8883	Nov. 20	-10	1 1	Co	-.494	-.563	4626	0457	- 29	+32	.2
.9237	Dec. 3	-11	3 4	Co	-.656	-.550	4562	0498	+ 5	-22	1
.9401	Dec. 9	-30	1 1	St	-.720	-.535	4520	0487	+ 32	-19	.5
.9702	Dec. 20	+35	4 8	Co	-.816	-.491	-9 4529	-2 0438	- 12	+13	1
.9811	Dec. 24	+28	4 4	Co	-.844	-.470	4491	0434	+ 14	+ 9	1
.9947	Dec. 29	+ 2	1 1	Co	-.872	-.441	4436	0423	+ 50	+11	.2
1.6849	Sep. 8	+ 6	2 4	Fr	+.618	-.148	1291	-1 9902	- 27	-16	.5
.7232	Sep. 22	+ 9	2 4	Fl	+.433	-.273	1246	9926	- 12	-26	.5
.7285	Sep. 24	+17	1 1	Fl	+.404	-.290	-9 1166	-1 9881	+ 65	+21	.2
.7367	Sep. 27	0	2 4	Fr	+.360	-.315	1265	9928	- 39	-25	.5
.7423	Sep. 29	+37	2 4	Fl	+.330	-.338	1220	9866	+ 2	+41	.5
.7450	Sep. 30	-10	2 4	Fr	+.315	-.338	1256	9870	- 35	+36	.5
.7352	Oct. 3	+ 9	2 4	Fl	+.269	-.361	1204	9903	+ 10	+ 5	1
.9879	Dec. 27	- 3	2 2	Co	-.860	-.457	-9 1111	-1 9861	- 82	- 2	.5
2.6541	Aug. 27	0	2 4	Fr	+.742	-.041	-8 9573	9582	- 11	+ 1	1
.6951	Sep. 11	- 4	2 4	Fr	+.572	-.183	9551	9613	- 27	-13	.5
.7443	Sep. 29	+10	2 3	Fr	+.319	-.338	9546	9689	- 59	-73	.5
.9490	Dec. 12	- 3	2 2	Fr	-.751	-.524	9392	9568	- 51	+25	.5
.9627	Dec. 17	+ 2	2 4	Fr	-.795	-.504	-8 9352	-1 9555	- 26	+30	1
.9682	Dec. 19	- 2	2 4	Fr	-.810	-.495	9310	9591	+ 10	-10	1
3.9622	Dec. 17	+29	1 2	Wy	-.793	-.505	7579	9276	+ 14	+19	.2
.9893	Dec. 27	-10	2 3	Fr	-.861	-.454	7538	9253	+ 16	+24	1
54.0195	Jan. 7	+24	2 4	Wy	-.906	-.381	7516	9211	0	+43	1
.7402	Sep. 28	+ 4	2 4	Wy	+.342	-.325	-8 5992	-1 9043	+ 31	-10	1
.7431	Sep. 29	+32	1 1	L	+.326	-.333	6054	9102	- 33	-66	.5
.8740	Nov. 15	- 5	2 3	Fr	-.419	-.560	5892	8991	+ 48	+49	.5
.9860	Dec. 26	+26	2 4	Wy	-.854	-.461	5842	8981	- 11	+ 8	1
55.7805	Oct. 12	+10	2 4	k	+.113	-.430	4273	8757	- 9	- 4	1
.9636	Dec. 18	+30	2 4	Wy	-.797	-.503	-8 4130	-1 8704	- 5	+ 9	1
56.6488	Aug. 25	- 3	2 4	Po	+.761	-.021	2605	8397	+ 32	+23	.5
.9515	Aug. 26	-35	1 1	Po	+.753	-.031	2686	8417	- 53	+ 4	.5
.8263	Oct. 28	- 8	2 4	Fr	-.155	-.516	2489	8426	+ 15	+39	1
57.6590	Aug. 29	-31	2 4	Fr	+.726	-.057	0863	8158	+ 15	-24	.5
58.6611	Aug. 30	+ 3	4 4	Fr	+.717	-.064	-7 9124	-1 7848	+ 30	- 3	1
.9506	Dec. 13	+17	2 4	Wy	-.756	-.522	9009	7896	- 70	-47	.5
59.0079	Jan. 3	+19	2 8	Fr	-.892	-.411	8848	7830	+ 18	-18	2

Bi = Leendert Binnendijk, Co = A. Wayne Conger, Dt = Daniel F. Detwiler, Fl = Robert Fleischer, Fr = Laurence W. Frederick, k = Peter van de Kamp, L = Sarah Lee Lippincott, Po = William Poole, Jr., St = Alden Stevenson, Wy = Arne A. Wyller.

and Lippincott 1949). The material consists of 8 exposures on 75 plates taken on 38 nights, representing total weight 26.5, and includes material used in the provisional parallax determination. A summary of the material is contained in Table I. The regular exposure time was 12 minutes. The plates were measured by the author on the Gaertner machine.

Table II gives information about the reference stars as well as the dependences for 1950.0 and

their annual changes. The positions of the standard frame and of L726-8 hold for the approxi-

TABLE II. REFERENCE STARS

No.	Diam.	m_p	α_0	δ_0	Dep.	$\Delta D/\text{yr.}$
	mm		mm	mm	1950	
1	.072	12.8	-44.67	-28.32	.407	-.00121
2	.111	11.9	+1.42	+19.29	.404	-.00159
3	.102	12.1	+43.25	+9.03	.189	+.00280
L726-8 A	.087	12.45	-9.43	-2.05		
B	.066	12.95				

mate epoch 1950. Due to the unfavorable observing conditions and the close separation of the components, the binary as a rule appears as a blended image which has been measured. On the nights of 1955 October 12 and 1956 October 28 the components are almost separated, and the images were measured both as a blend and as a separated double star. The position of the blend was represented formally as a weighted mean position of the components, i.e., it could be considered the photocenter of the components for a light ratio of 0.60 to 0.40 or a magnitude difference of 0.5, close to the observed photovisual magnitude difference of the components.

On the plates taken on 1958 August 30 and 1959 January 3, the components are clearly separated and yield the following relative positions:

Epoch	θ	ρ
1958.66	38°.2	(2".01)
59.01	34.9	(1.84)

The position angles may be trusted but the separations are probably in need of a correction of about +".4, judging from earlier experience with Krüger 60 (van de Kamp 1937). Assuming the above light ratio, the measured positions of the components on these nights were reduced to a weighted mean "photocenter" to be included in the parallax solution.

The majority of plates are of fair quality only, principally due to the fuzzy quality of the exposures obtained at this low declination; hence the large number of nights with fractional weights. Although the faint component of L726-8, UV Ceti, is a well-known flare star, there is no evidence of flares on any of the photographs in the present parallax series.

The nightly means were represented by the following equations of conditions:

$$X = c_x + \mu_x t + \pi P_\alpha$$

$$Y = c_y + \mu_y t + \pi P_\delta,$$

where the symbols have the usual meaning; t is counted from 1960.000. A combined solution for both coordinates gives the following results, the scale factor being 1 mm = 18".87.

mm	p. e.	weight
$c_x = -7.69724$		
$c_y = -1.74435$		
$\mu_x = +.173343 = +3".2710 \pm ".0023$	220.54	
$\mu_y = +.029027 = +.5477 \pm ".0023$	220.46	
$\pi = +.019543 = +.3688 \pm ".0098$	11.93	

$$\text{p.e. } \pi = \pm .001787 = \pm .0337$$

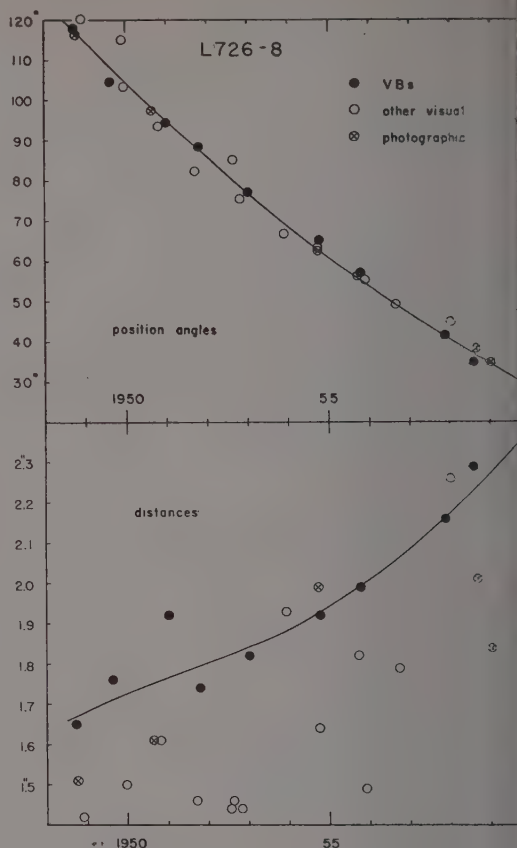


Figure 1. L726-8: Position angles and distances; observed and computed.

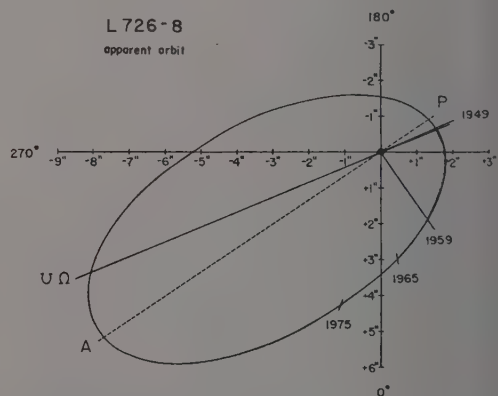


Figure 2. L726-8: Apparent orbit, based on observations covering the sector 1949-59.

The residuals from this solution are given in Table I. The weighted mean residuals for normal places are given in Table III, together with their

TABLE III. NORMAL PLACES OF RESIDUALS

Epoch	Σp	Σn	v_x unit .0001	v_y mm
1949.87	4.6	8	+21	-10
51.76	4.2	8	-18	+3
52.84	4.5	6	-21	-2
54.49	5.2	7	+9	+12
56.70	5.0	7	-7	+4

mean epoch, total weight, and number of nights. The nights of 1958 August 30 and 1959 January 3, for which the photocenter has been calculated from the measured positions of the components, are not included in this summary. The material is not sufficient to permit a determination of the mass ratio; it does not, however, indicate any striking disparity between the masses.

Orbital motion. Recent micrometer observations, kindly furnished by Dr. Van Biesbroeck and from the Lick card catalogue by Dr. Jeffers, indicate a marked increase in separation, with a corresponding decrease in angular motion. These observations are no longer satisfied by the orbital dimensions given by Luyten (1954) nor by the provisional orbit of Protitch (1955). An attempt was thus made to compute a new orbit.

The measuring difficulties of this faint object are mentioned by several observers (van den Bos 1951; Van Biesbroeck 1954; Protitch 1955; Markowitz 1956). Appreciable systematic errors are clearly present in the distance measures, both visual and photographic. It is only through the continuous series of observations by Van Biesbroeck that an analysis appears at all possible.

The short arc covered by the observations obviously still permits only a provisional orbit calculation. No trouble was encountered with the position angles but considerable adjustment of the distances was required to satisfy the law of areas. The adjusted smooth curves for θ and ρ were analyzed by Russell's (1917, 1933) elegant and effective method.* Various trials indicate that although the observations could be represented by a parabolic orbit, the best representation at this stage is reached by an orbit with period 200 years, eccentricity 0.7 and periastron passage 1948.1. With these dynamical elements, the Thiele-Innes constants were obtained from the yearly points read from the empirical curves

for θ and ρ ; the calculated curves show only slight and permissible deviations from the empirical curves. The resulting elements are summarized as follows:

$$\begin{aligned}
 P &= 200 \text{ years} \\
 e &= 0.7 \\
 T &= 1948.1 \\
 B &= +4''.542 \\
 A &= -3.055 \\
 G &= +2.869 \\
 F &= +3.037
 \end{aligned}
 \quad \text{or} \quad
 \begin{aligned}
 a &= 5''.57 \\
 i &= 136''.7 \\
 \Omega &= 112''.5 \\
 \omega &= 344''.5
 \end{aligned}$$

Epoch	θ	ρ
1960.0	28''.9	2''.38
61.0	24.2	2.50
62.0	19.8	2.62
63.0	15.7	2.75
64.0	12.0	2.88
65.0	8.7	3.01

The position angles of all observations are satisfactorily represented by this orbit, while the distances of visual observers, other than Van Biesbroeck, generally are too small by appreciable amounts up to 0''.5. The same holds for the photographic measures (Figures 1 and 2).

It appears to be a stroke of good luck that the observations started about periastron, the first decade of observations obviously being critical, showing considerable curvature. The next few decades can add only little toward an improved orbit. The star is going to be an easy object for some time to come; the present orbit indicates a maximum separation of over 9'' near apastron!

Dynamical interpretation; mass-luminosity relation. The present parallax determination, $+".369 \pm ".010$, agrees closely with Strand's value of $+".380 \pm ".011$ (Luyten 1956). Assuming an absolute parallax of $".38$, the sum of the masses is $0.079 \odot$ —exactly the same value as Luyten's, although quite different orbital elements were used. This coincidence is explained by the use of the same parallax and the observations having been made close to the positions of the node, so that the curvatures represented by different orbits do not appreciably differ (Russell 1917). With the present accuracy of the parallax, and the well determined curvature of the orbit over the past decade, it is not likely that there will be a substantial change in the calculated sum of the masses in the near future.

L726-8 furnishes a substantial extension of the fainter end of the mass-luminosity relation beyond Krü 60 and Ross 614. The mass-luminosity relation for M-dwarfs is virtually linear, the bolometric magnitude decreasing 0.5 for a

* There is an error in formula (4) in Russell's article (1933). Instead of $1/2 bX$ read bX .

TABLE IV. MASSES AND LUMINOSITIES OF DWARF-M COMPONENTS OF VISUAL BINARIES

Name	Apparent visual magnitude	Spectrum	Absolute visual magnitude	Absolute bolometric magnitude	Mass	Log mass
Fu 46 A	10.01	M4	10.96	8.72	.31☉	— .51
B	10.39	M4	11.34	9.10	.25	— .60
Krü 60 A	9.82	dM4	11.84	9.60	.272	— .57
B	11.37	dM6	13.39	10.58	.164	— .79
♂ Eri C	11.10	M5e	12.62	10.10	.21	— .68
Ross 614 A	11.34	dM6+	13.34	10.53	.14	— .85
B	14.8		16.8	12.3	.08	— 1.10
L726-8 A	12.45	dM5.5e	15.35	12.68	.044	— 1.36
B	12.95	dM5.5e	15.85	13.18	.035	— 1.46

decrease of 0.1 in the logarithm of the mass (van de Kamp 1954a, 1958). There seems to be little objection to adopting this ratio for the components of L726-8; on the basis of their magnitude difference 0.5, their mass ratio would then be 1.26 and the individual masses $M_A = 0.044☉$, $M_B = 0.035☉$.

The best available data for mass and luminosity of M-dwarfs are collected in Table IV. For stars other than L726-8, the observing data have been derived from an earlier tabulation by the author (van de Kamp 1958), making use of certain new data given by Limber (1958). The bolometric magnitudes for Krü 60 A, B are those given by Limber; for the other stars, the bolometric corrections correspond to the spectral

type using Limber's data for Krü 60 A, B standards. For Ross 614B, whose spectral type is unknown, the bolometric correction (-4.1 was based on the absolute magnitude (Limber Figure 3). The results for L726-8 appear to confirm the general trend of the mass-luminosity relation for M-dwarfs established thus far.

The mass-luminosity relation for M-dwarfs plotted in Figure 3 shows a remarkably small dispersion which may prove to be even smaller when improved values of the bolometric corrections become available.

REFERENCES

- Joy, A. H. and Humason, M. L. 1949, *Pub. A. S. P.* 6, 133.
 Limber, D. N. 1958, *Ap. J.* 127, 353.
 Lippincott, S. L. 1951, *A. J.* 55, 236.
 —. 1955a, *Sky and Telescope* 14, 364.
 —. 1955b, *A. J.* 60, 379.
 Luyten, W. J. 1949, *Ap. J.* 109, 532.
 —. 1950, *Pub. A. S. P.* 62, 274.
 —. 1954, *Ibid.* 66, 337.
 —. 1956, *Ibid.* 68, 258.
 Markowitz, Wm. 1956, *Pub. U. S. Naval Obs.* 17, part 194.
 Protitch, M. B. 1955, *Bul. Obs. Astr. Beograd* 19, No. 11.
 Russell, H. N. 1917, *A. J.* 30, 131.
 —. 1933, *M. N.* 93, 599.
 Van Biesbroeck, G. 1954, *Pub. Yerkes Obs.* 8, part VI.
 van de Kamp, P. 1937, *A. J.* 47, 1.
 —. 1954a, *Amer. Sci.* 42, No. 4, 572.
 —. 1954b, *A. J.* 59, 447.
 —. 1958, *Handbuch der Physik* (Berlin: Springer) 5, 187.
 van de Kamp, P. and Lippincott, S. L. 1949, *A. J.* 55, 1.
 —. 1950, *Pub. A. S. P.* 62, 47.
 van den Bos, W. H. 1951, *Union Obs. Circ.* 111, 13.
 Wagman, N. E. 1956, *A. J.* 61, 191.

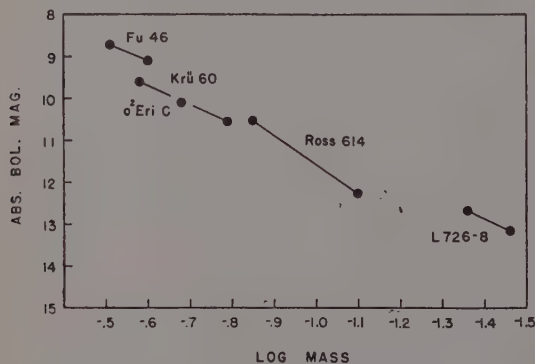


Figure 3. Mass-luminosity relation for dwarf-M components of visual binaries; the mass-ratio for L726-8 has been assumed as 1.26.

SIX VARIABLE STARS OF UNUSUAL TYPE IN SAGITTARIUS

By DORRIT HOFFLEIT

Maria Mitchell Observatory, Nantucket, Massachusetts

Received May 29, 1959

Abstract. The results of photographic observations on Harvard and Nantucket plates are given for six peculiar variables in VSF 193. Three of them belong to the R Coronae Borealis class. As ten per cent of all the known variables of this type have been found in VSF 193 (four stars) and twenty per cent in the whole of the constellation Sagittarius, a high concentration toward the galactic center is inferred. One probable new flare star is announced. The other two objects are apparently semi-regular. A previous provisional period for GW Sgr has not been confirmed.

In as rich and complex a star field as VSF 193 in Sagittarius it might be expected that numerous variables of the unusual types would be discovered. Six, listed in Table I, are discussed here. Two of them were previously discovered by Luyten (1927) and one by Miss Woods (1928) at Harvard; three, for which charts are given in Figure 1, are new discoveries heretofore reported

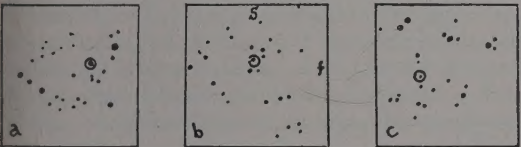


Figure 1. Environment of the new variables in Table I. Width of charts, 15' arc.

only in abstract (Hoffleit 1958a). Of the six variables, three are apparently examples of the R Coronae Borealis type, one is probably a flare star, while the other two have semi-regular variations and may belong to the RV Tauri and RW Aurigae types. They have all been examined on at least 200 plates of the Harvard A, B and MF series taken between 1924 and 1951 with the 24-inch Bruce, 8-inch Bache and 10-inch Metcalf refractors, respectively. All but the faintest have been examined on additional plates of the Harvard patrol series and on more recent photographs taken with the 7.5-inch Cooke triplet at the Maria Mitchell Observatory. In Table I the column headed *Obs.* gives the approximate total

number of photographs used and the initials of the persons making the magnitude estimates: A, Mrs. Jean Hales Andersen; F, Margo Friedel; S, Leo Schneider; and H, myself.

Observations of the first star in Table I are plotted in the top strip of Figure 2. Only two maxima are revealed, one represented by a single observation at magnitude 15.0 on J.D. 2424688, the other by 8 observations from J.D. 25475 to 25830. At maximum, fluctuations of about one magnitude have been found. The star is probably of the R Coronae Borealis type.

GU Sgr was discovered in 1927 by Luyten who ascertained the type of variation and published the light curve for the years 1889 to 1926. The light curve from over 600 Harvard and Nantucket plates taken since 1924 is given in Figure 2. Three groups of minima are found, representing more complex behavior than had previously been noted.

The third R Coronae Borealis type star, MV Sgr, had been found by Miss Woods (1928) but she published only the magnitude range from a few observations without noting the type of light variation. The recent observations, plotted in Figure 3, show two groups of minima. Because of his interest in R Coronae Borealis type stars, I called MV Sgr to the attention of Dr. George H. Herbig at Lick Observatory. He reports (private communication) that two spectrograms he obtained near maximum light indicate that the star has an extraordinary spectrum, probably an early

TABLE I. SOME PECULIAR VARIABLES IN SAGITTARIUS

Designation Sgr	Discoverer	R.A. (1900)	Dec.	Max.	Min.	Sp.	Obs.	Type
a	Hoffleit	18 ^h 15 ^m 20 ^s	-24° 47' 8"	13.5	15.5	—	380 H	R Cor Bor
GU	Luyten	18 08	-24 18.2	11.3	15.0	—	600 A, S	R Cor Bor
GW	Luyten	19 16	-25 05.5	13.8	15.6	—	250 F, H	RV Tau? Per. 119 days
b	Hoffleit	31 37	-20 58.3	12.1	15.7	B?	550 A	RW Aur? Semi-regular periods of 5000? and 200-400 days
c	Hoffleit	32 44	-21 48.7	12.6	15.	K5	700 A	Flare
MV	Woods	38 34	-21 03.3	12.0	15.6	B?	550 A, F	R Cor Bor

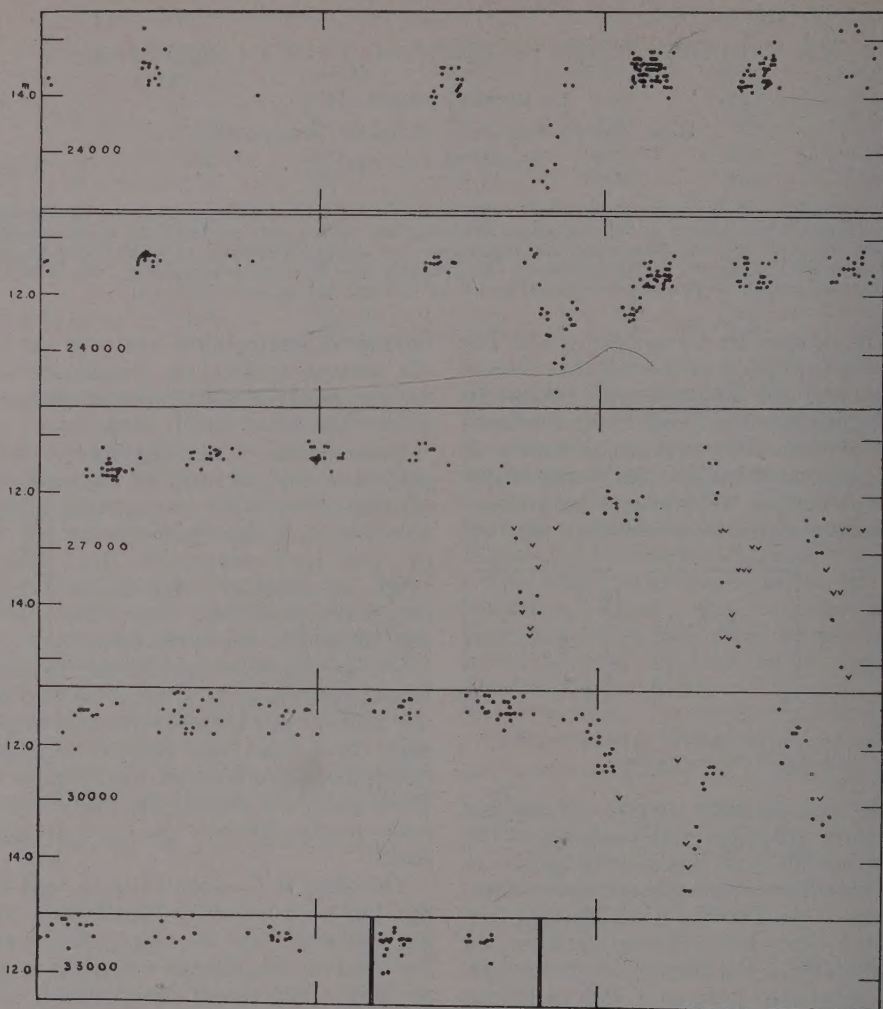


Figure 2. Two stars of R Coronae Borealis type. Top strip, Variable *a*. The other strips represent GU Sgr between J.D. 2424000 and 2434000 and, inset, J.D. 2436000–36600. Markers indicate intervals of one magnitude and 1000 days. J.D. —2400000 of beginning of each strip noted at left.

B-type, showing at low dispersion nothing but absorption lines of *HeI*. Herbig adds, "The only other spectrum I know like this is Popper's (1942; 1946; 1947) helium star, H.D. 124448, which has a counterpart found by Thackeray. But neither of those is known to be variable." In view of the long duration at maximum of MV Sgr, it would not be surprising if the variability of other similar stars had failed to be discovered.

The 1958 edition of the General Catalogue of Variable Stars lists a total of 39 stars of the R Coronae Borealis type, six of them in Sagit-

tarius. Only one of the three listed in Table I was previously recognized as belonging to this class. Thus 8 in 41, or nearly twenty per cent of the stars of this class, are concentrated in the one constellation, Sagittarius; and of these four, including V348 Sgr discussed earlier (Herbig 1958; Hoffleit 1958b), are in the 80 square-degree area of VSF 193. There may well be a few others; as yet, among the several hundred known and more than 300 unpublished variables in this field, only the stars of known large amplitude have been adequately examined for the determination of types.

A star of exceptional interest and uncertainty as to type is GW Sgr, discovered by Luyten (1927). On the basis of 140 magnitude estimates for the years prior to 1926, Luyten deduced that the star is usually at maximum and may be an eclipsing variable of extraordinarily long period. He derived a provisional period of 732 days, almost exactly two years, and recorded well-defined minima near J.D. 23622 and 24375. Luyten commented that the star could conceivably be semi-regular or of the R Coronae Borealis types. Margo Friedel and I examined the star on about 250 Harvard and Nantucket plates taken between 1924 and 1958 but could not confirm the provisional published period. We found six intervals between minima indicating a period close to or a submultiple of about 360 days. From this material Miss Friedel derived a period of 179 days fitting the vast majority of our observations. However, a night run of ten plates taken on July 30, 1951, J.D. 33858, showing the star at maximum light, falls precisely over the interval of computed minimum. I then found a moderately satisfactory period of 119 days fitting all of our observations, as shown in Figure 4. But neither of the two periods satisfies the minimum observed by Luyten on J.D. 23621-2, only a few

epochs prior to the date of our earliest observation, J.D. 23908. The two computed periods are stroboscopically related $\left(\frac{1}{179} + \frac{1}{365} \approx \frac{1}{119}\right)$ and

we may have simply a spurious period similarly related to the true period through the seasonal gaps in the observations. Probably the star is semi-regular, possibly of the RV Tauri type.

In Figure 5 the observations of the fourth variable in Table I, star *b*, show an apparent superposition of different cycles of variation. There are two major cycles of about 5000 days having an overall amplitude of nearly four magnitudes. At the same time, variations of the order of a magnitude or more occur in cycles of 200 to 400 days. In this respect the star appears to resemble the peculiar long-period variable, V Hydrae, which has an N-type spectrum. Dr. Herbig kindly offered also to observe this variable spectroscopically. On July 20, 1958, he found its spectrum peculiar, "the continuum being like that of a hot star with *H*-beta emission. What little can be seen of the absorption spectrum looks a little like MV Sgr." Thus it is more likely that the star belongs to the peculiar RW Aurigae than the long-period class.

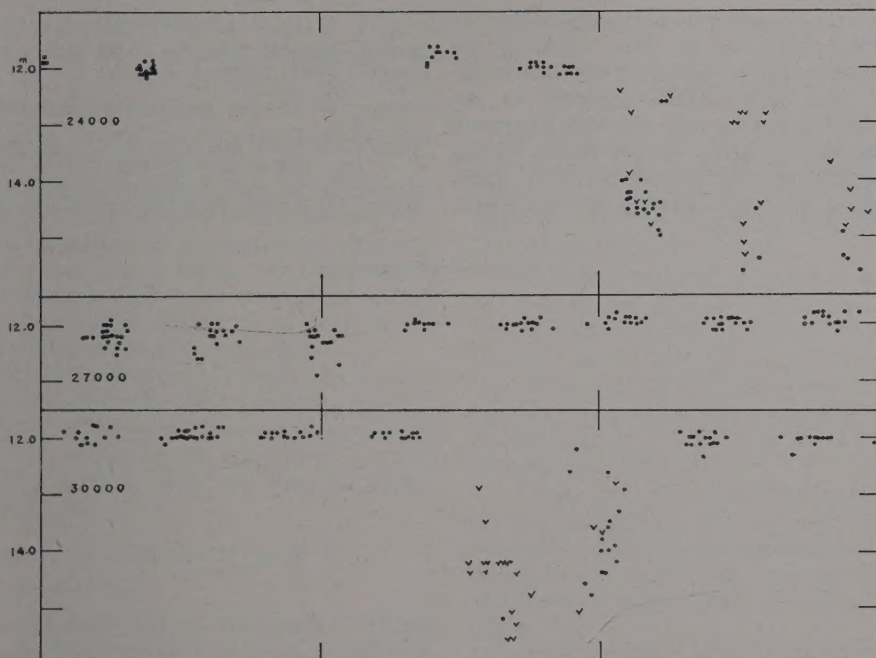


Figure 3. Observations of MV Sgr. Markers indicate intervals of one magnitude and 1000 days. At left, J.D. -2400000 of beginning of each strip.

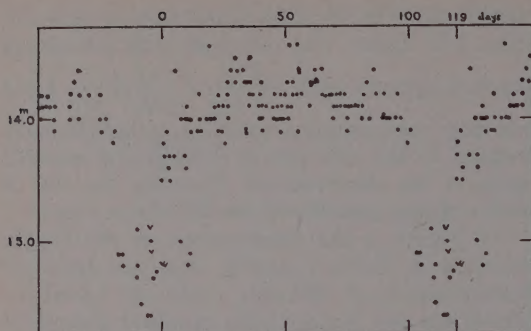


Figure 4. The observations of GW Sgr since 1924 fitted to a period of 119 days.

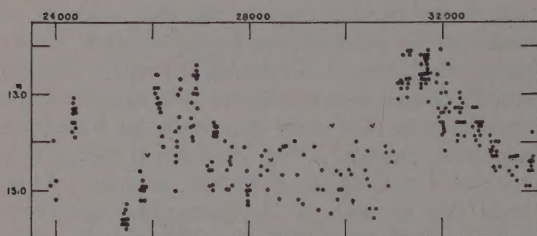


Figure 5. Observations of Variable b. Note that markers are at 2000-day intervals.

Finally, star *c* appears to be a flare star. It was carefully examined by Mrs. Andersen on nearly 700 Harvard plates. Only the one maximum has been found which occurred on the discovery plates. Fortunately the flare happened on a night when a series of five plates on the region was obtained, on August 21, 1928. Table II gives the magnitude estimates for that night.

Normally the star is between 14.5 and 15.0 photographic magnitude. Dr. Herbig obtained its spectrum on July 14, 1958, finding it to be K5 without emission. At the time he estimated the magnitude as about 16 visual.

TABLE II. OBSERVATIONS OF THE FLARE OF VARIABLE

J.D	<i>m</i> _{pg}
2425480.222	14.0
.253	14.2
.285	13.6
.317	12.6
.348	12.9

A large percentage of the magnitude estimates discussed here, especially those by Mrs. Andersen, was obtained about two years ago under grant from the National Science Foundation. Miss Friedel's work as a student assistant at the Maria Mitchell Observatory in the summer of 1958 was supported by a grant from the Goul Fund of the National Academy of Sciences. Mr. Schneider was a volunteer assistant. For all the help it is a pleasure to express my appreciation to Dr. George H. Herbig of Lick Observatory. I am particularly indebted for his interest in studying the spectra of our peculiar variable stars.

REFERENCES

- Herbig, George H. 1958, *Ap. J.* **127**, 312.
 Hoffleit, Dorrit. 1958a, *A. J.* **63**, 50.
 —. 1958b, *ibid.*, 78.
 Luyten, W. J. 1927, *Bull. Harvard Coll. Obs.* No. 85, 4 and 6.
 Popper, Daniel M. 1942, *Pub. A. S. P.* **54**, 160.
 —. 1946, *Pub. A. S. P.* **58**, 370.
 —. 1947, *Pub. A. S. P.* **59**, 320.
 Woods, Ida E. 1928, *Bull. Harvard Coll. Obs.* No. 855, 2